Computation of Effective Number of Bits, Signal to Noise Ratio, & Signal to Noise & Distortion Ratio Using FFT — *APPLICATION NOTE*

February, 2011

Summary

This application notes discusses how to quantify errors in sampled data systems using several figures of merit based on frequency domain measurements

Introduction

Sampled data systems are expected to produce data that is a faithful reproduction of a continuous input signal. If perfect, the sequence of data samples would exactly represent the voltage of the waveform sampled at the points in time where the samples are taken. A perfect sampled data system is not possible due to a variety of noise and distortion factors discussed in more detail below.

An imperfect sampling system produces a sequence of data that does not exactly represent the sampled analog waveform. The difference between the actual waveform and the input waveform is called the error signal. The power content in the error signal causes degradation of the sampled signal.

When looked at in the time domain, it is often difficult to distinguish between the error signal and the input signal. Therefore, frequency domain methods are often used, usually using a spectrum analyzer. When a sinusoidal input is applied to a system and examined in the frequency domain, it is easy to distinguish the error from the input. This is because the input signal should represent a spectral peak at a single frequency. All other spectral components are considered as error.

Metrics have been established to quantify the quality of sampling systems. These metrics tend to separate the sources of error such that examination of such metrics provide indications of the causes for degradation in a system. This paper is concerned with the metrics based on frequency domain measurements. Since many of these metrics are based on power measurements of frequency related components, this paper also addresses these issues. Specifically, it addresses these measurements performed using the Discrete Fourier Transform (DFT) applied to the data samples.

Errors in Sampled Data Systems

There are several sources of error in sampled systems. These sources are, in the simplest case, broken down into noise and distortion.

Distortion

Distortion is considered as error in the acquired waveform that has a high degree of correlation with the signal. In other words, distortion is not random, but is dependent in some manner on the input.

For the purpose of this paper, distortion is defined as follows:

> *Distortion is defined as any portion of the error signal whose frequency locations are functions of the input frequency.*

Knowing the input frequency, the distortion frequencies can always be calculated if the mathematical model for the distortion is known. For example, the most common form of distortion is harmonic distortion. With harmonic distortion, the distortion components appear at integer multiples of the input frequency. Typical sources of harmonic distortion are non-linearity of the transfer function of the system including saturation, clipping, slew-rate limiting and others.

Other forms of distortion are possible. Often these forms are known based on the design of the system.

Noise

Noise, as opposed to distortion, is assumed to be uncorrelated with the input. For the purpose of this paper, noise is defined as follows:

> *Noise is defined as any portion of the error signal whose frequency locations are not functions of the input frequency.*

Noise itself is commonly broken into categories depending on two characteristics:

1. The distribution of the noise (i.e. the shape of the histogram of the error).
2. The frequency domain shape of the spectral density (i.e. the shape of the noise plotted as a function of frequency).

When categorized based on frequency domain shape, noise that is evenly spread across all frequencies is called "white" noise. Noise that is distributed such that the noise power in each octave is constant is called "pink" noise. There are many other noise shapes.

Noise is also categorized by distribution. Noise whose distribution is normal is called gaussian noise. There are many sources of gaussian noise. Noise is also created through quantization - effectively the round-off error in converting analog voltages to digital numbers. The simplest quantization methods produce a uniform error distribution that is white. Many methods of quantization do not produce uniformly distributed white noise.

Error Classification

The analysis of error in the system reduces to measuring and separating the input signal from the distortion and noise. Often the offset error is treated separately, as well. Furthermore, the distortion and noise are often further classified, because different distortion or noise sources involve different problems in the physical design of the system.

For the purpose of this paper, we will be concerned with the power content of a sinusoidal input signal. The power content of the harmonics will be considered as distortion, while all other power content will be considered as noise.

Figure 1 illustrates the breakdown of components of an acquired signal and how these components are categorized for the purpose of the measurements subsequently explained:

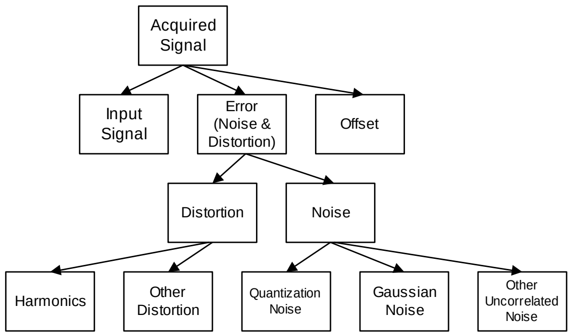


Figure 1: Categorization of component power levels

Figures of Merit

The previous section explained the classification of error in a sampled system. In order to quantitatively determine the effectiveness of a system, we must use common sets of measurements. These measurements are called "figures of merit". These are measurements for which there are common definitions such that numerical values instantly convey the quality of the system. These measurements are explained briefly in Table 1. You should examine these definitions and understand their relationship to error sources as shown in Figure 1. There are many other figures of merit for systems. Some are well known and others depend on the system under analysis. When necessary, a definition of the figure of merit must accompany the calculated value for any sense to be made of the measurement.

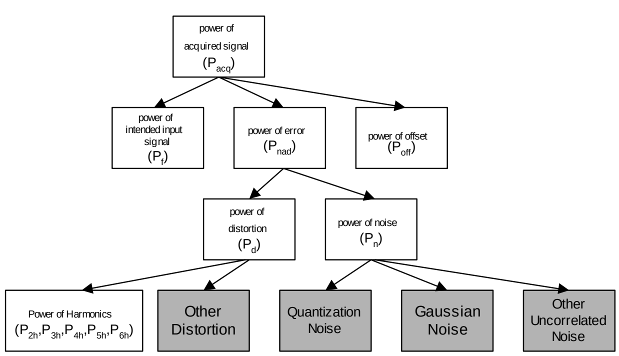


Figure 2: Categorization of Noise Powers

Abbreviation	Name	Description	Formula
SINAD	Signal to noise and distortion ratio	The ratio of the RMS signal to the RMS sum of all other spectral components, including the harmonics except DC. Usually expressed in dB. The more positive the value, the better the system.	$SINAD = P_{f(dB)} - P_{nad(dB)}$
ENOB	Effective number of bits	SINAD can be expressed directly as ENOB - they are equivalent. ENOB is the number of bits in a quantizer that would produce the exact same SINAD value in a system whose entire source of noise is quantization itself.	$ENOB = \frac{(SINAD - 1.76)}{6.02}$ see the section at the end of this document entitled "Derivation of the ENOB Equation" for an explanation of this equation.
SNR	Signal to noise ratio	The ratio, expressed in dB, of the RMS signal amplitude to the RMS sum of all other spectral components excluding the distortion and offset error	$SNR = P_{f(dB)} - P_{n(dB)}$
THD	Total harmonic distortion	The ratio, expressed in dBc, of the RMS sum of the first five harmonic components, to the RMS value of the measured fundamental component.	$THD = P_{d(dB)} - P_{f(dB)}$

Table 1: Some Figures of Merit For Sampled Systems

The generation of the figures of merit in Table 1 reduces to the evaluation of the equations set forth which in turn requires the calculation of the power contained in particular spectral components in an acquired signal as shown in Figure 2. The remainder of the document will be concerned with describing the steps necessary to perform these measurements using the DFT of a signal acquisition.

Figure of Merit Calculations Using the DFT

The steps required in calculating figures of merit using the DFT are as follows:

1. Acquire a signal.
2. Calculate the DFT of the waveform.
3. Normalize the DFT such that it is suitable for power measurements.
4. Identify the DFT bins that contain the powers of the elements shown in Figure 2.
5. Calculate the total power of these components
6. Calculate the figures of merit using the powers calculated above.

Acquisition of the signal

The continuous analog signal is sampled for some duration using a digitizer such that the resulting acquisition is an array of numbers whose value corresponds to the voltage of the waveform and whose array index corresponds to a time. In other words, an array x of length K results such that:

$$k \in 0 \ldots K-1$$
$$v(t = k \cdot T) = x_k$$

Equation 1

Calculating the DFT

It is well known that the FFT is simply a fast computation method for the Discrete Fourier Transform (DFT). The generally accepted definition of the DFT is given by:

$$X_n = \frac{1}{K} \cdot \sum_{k=0}^{K-1} x_k \cdot e^{-j \cdot \frac{2 \cdot \pi \cdot n \cdot k}{K}}$$

Equation 2 - Definition of the DFT

The result of the FFT is an array of complex numbers whose magnitude corresponds (with two slight exceptions) to 1/2 of the amplitude and whose argument corresponds to the phase of a cosine frequency component of the waveform and whose array index corresponds to a frequency. In other words, an array X of length $\frac{K}{2}$ such that for each n:

$$n \in 0 \ldots \frac{K}{2}$$
$$A\left(f = \frac{n}{K} \cdot F_s\right) = 2 \cdot |X_n|$$
$$\theta\left(f = \frac{n}{K} \cdot F_s\right) = \arg(X_n)$$

Equation 3

The exception is DC and Nyquist where the actual amplitude is:

$$A(f = 0) = |X_n|$$
$$A\left(f = \frac{K}{2}\right) = |X_n|$$

Equation 4

Each complex number X_n is considered the value in a frequency "bin". The frequency of a bin and the bin location of a particular frequency are defined as follows:

$$f(n) = \frac{n}{K} \cdot F_s$$

Equation 5

$$n(f) = K \cdot \frac{f}{F_s}$$

Equation 6

Where F_S is the sampling frequency

Before calculating the FFT, the acquisition should be windowed. This is because the frequency of the input signal will not generally be known exactly. Because of this inexactness, the input sinusoid will not necessarily be continuous at the endpoints of the acquired waveform. Without windowing, spectral spreading due to these discontinuities tends to spread the power of the input sinusoid over all of the frequencies. Note that windowing itself causes both concentration of the power of the input signal and some slight spreading of the spectrum. This spreading will be accounted for by summing the power *about* a particular frequency, not *at* a particular frequency, which takes this slight spectral spreading into account.

Windowing of the signal involves the point-by-point multiplication of the acquired waveform by some function (like a raised cosine, for example). The function for the raised cosine is:

$$w_k = \frac{1}{2} - \frac{1}{2} \cdot \cos(2 \cdot \pi \cdot \frac{k}{K})$$

Equation 7

The windowed acquisition is:

$$x_k \rightarrow x_k \cdot w_k$$

Equation 8

Windowing will affect the absolute values of the power calculations. Furthermore, while windowing tends to concentrate the power of the input sinusoid, it always produces some slight spreading of the spectrum, sometimes only by a few FFT bins. The power calculations must be performed by summing the power around the frequencies of interest. Additionally, the resultant sum will need to be corrected to account for windowing. This is explained later in the document.

Normalization of the FFT

The FFT in itself is not readily usable for power calculations. The spectral components generated by the FFT need to be normalized first such that each component contains a value representing its Fourier series amplitude and phase. After this normalization, the signal could be reconstructed by summing Cosines at each frequency with the amplitude and phase given.

It is obvious from Equation 3 and Equation 4 that in order to normalize the DFT, each value in X (except for the DC value) needs to be multiplied by 2.

Unfortunately, some algorithms for the FFT do not produce exactly the same results as the DFT definition shown in Equation 2..

MathCAD, for example, uses several non-standard forms of the FFT which causes quite a bit of confusion. For example, the cfft( ) function is defined as:

$$X_n = \frac{1}{\sqrt{K}} \cdot \sum_{k=0}^{K-1} x_k \cdot e^{j \cdot \frac{2 \cdot \pi \cdot n \cdot k}{K}}$$

Equation 9 - MathCAD's definition of complex FFT (cfft)

Obviously, a different method of normalization will be necessary with this definition of the FFT.[1]

Table 2 shows how to convert between the standard definition of the DFT, MathCAD's cfft( ), and Amplitude/phase (Fourier component normalized).

[1] I've noticed that in the later versions of MathCAD, an alternate version of the FFT called CFFT( ) has been provided which has the same form as the standard definition of the DFT which I've provided - All confusion can be removed by simply using the CFFT( ) function.

		From	
To	**DFT**	**MathCAD cfft()**	
DFT		$X_n \to \left(\frac{1}{\sqrt{K}} \cdot X_n\right)^*$	$X_n = \begin{cases} A_n \cdot e^{j \cdot \theta_n} & if \quad n = 0 \\ A_n \cdot e^{j \cdot \theta_n} & if \quad n = \frac{K}{2} \\ \frac{A_n}{2} \cdot e^{j \cdot \theta_n} & otherwise \end{cases}$
MathCAD cfft()	$X_n \to \left(\frac{K}{\sqrt{K}} \cdot X_n\right)^*$		$X_n = \begin{cases} A_n \cdot \sqrt{K} \cdot e^{-j \cdot \theta_n} & if \quad n = 0 \\ A_n \cdot \sqrt{K} \cdot e^{-j \cdot \theta_n} & if \quad n = \frac{K}{2} \\ A_n \cdot \frac{\sqrt{K}}{2} \cdot e^{-j \cdot \theta_n} & otherwise \end{cases}$
Amplitude-phase	$A_n = \begin{cases} \lvert X_n \rvert & if \quad n = 0 \\ \lvert X_n \rvert & if \quad n = \frac{K}{2} \\ 2 \cdot \lvert X_n \rvert & otherwise \end{cases}$ $\theta_n = \arg(X_n)$	$A_n = \begin{cases} \frac{1}{\sqrt{K}} \cdot \lvert X_n \rvert & if \quad n = 0 \\ \frac{1}{\sqrt{K}} \cdot \lvert X_n \rvert & if \quad n = \frac{K}{2} \\ \frac{2}{\sqrt{K}} \cdot \lvert X_n \rvert & otherwise \end{cases}$ $\theta_n = -\arg(X_n)$	

Table 2: Table for Fourier Component Normalization from FFT

In practice, only the magnitude (or amplitude) information is necessary for power measurements. Furthermore, for power measurements, it is best to convert this information further into rms or dB.

The reasons for this are as follows:

1. Amplitude needs special handling at DC and Nyquist when power calculations are concerned. As an example, a DC voltage with amplitude of 1 V has an rms voltage of 1 V. However a sinusoidal signal which amplitude of 1 V has an rms voltage of 707.1 mV. Converting to rms or dB removes the need for special handling of DC down the line.
2. In order to calculate the power in spectral components, the power of multiple frequencies are summed. The powers to not simply add. For example, most everyone is familiar with the fact that rms voltages sum as the root sum of the squares, so the determination of the rms voltage is conceptually easy.
3. The end result of most of the measurements is power ratios, which are most conveniently expressed in dB.

Table 5 and Table 6, at the end of this document, contains formulas for conversion between rms, amplitude, peak-peak, and dB.

In order to use Table 5 to convert between dB, you must first have some information on hand depending on the industry in which you are working. Basically, you need to know the value of the resistance where the voltage is measured across. Generally, this resistance is the most common value of termination resistor. You will also need to know the reference power.

Here is a table of this information for several industries:

	Industry		
	Radio Frequency	**Audio**	**TV**
Load Resistance (R)	50 Ω	600 Ω	75 Ω
Definition of 0 dB	1 mW into 50 Ω	1 mW into 600 Ω	1 mV rms across 75 Ω
P_{ref}	1 mW	1 mW	13.333 nW
$10 \cdot \log(R)$	16.990	27.782	18.751
$10 \cdot \log(P_{ref})$	-30	-30	-78.751

Table 3 - Load Resistance and Reference Power Values

Table 6 contains conversion formulas with numbers already provided for RF.

Table 5 and Table 6 should be used to convert each component value in the FFT to rms, or decibels so that it can be used directly for power calculations.

At this time, the correction for windowing of the FFT should be applied. The correction is multiplicative if the correction is applied to any voltage units except for dB. For dB, the correction is additive. The correction is calculated as follows:

$$wc_{dB} = -20 \cdot \log\left(\sqrt{\frac{1}{K} \cdot \sum_k w_k{}^2}\right)$$

Equation 10 – additive windowing correction for decibels

$$wc = \frac{1}{\sqrt{\frac{1}{K} \cdot \sum_k w_k^{\ 2}}}$$

Equation 11 – multiplicative windowing correction

Frequency Component Power Calculations

There are a few steps to the process of calculating frequency component power levels:

1. Identify the frequencies of the components you need to calculate the power of. At the minimum, you will need the fundamental (the input sinusoid), and you will probably need the harmonics.

Component	Frequency
Fundamental (sometimes called the first harmonic)	f_0
Second harmonic	$2 \cdot f_0$
Third harmonic	$3 \cdot f_0$
Fourth harmonic	$4 \cdot f_0$
Fifth harmonic	$5 \cdot f_0$

Calculate the FFT index of these components using

2. Equation 6. Since you will want the best integer bin value, use:

$$n(f) = floor\left(K \cdot \frac{f}{F_s} + .5 \right)$$

Equation 12

3. Determine the frequency range (f +/- Δf) of the component. Remember that although you will use an input sinusoid at a particular frequency, the power of the signal will be spread over several FFT bins. This is due to jitter, leakage in the FFT, modulation, etc. and most importantly, windowing. You should determine the frequency range to use by examining the FFT. Furthermore, you should take a large enough FFT such that the frequency range can be made fairly large without affecting the integrity of your measurement. This is shown clearly in the example provided.

4. Determine the FFT bin range using the following equations. Use these to calculate the beginning and ending bin of a component at a particular frequency (f) with a frequency spread (Δf):

$$n_0(f, \Delta f) = floor\left(K \cdot \frac{f + \Delta f}{F_s} + .5 \right)$$

Equation 13

$$n_f(f, \Delta f) = floor\left(K \cdot \frac{f - \Delta f}{F_s} + .5 \right)$$

Equation 14

5. Sum the normalized FFT components from n_0 to n_f. Remember that you cannot simply add the values. For example, if the normalized FFT components are in rms voltage, they add as the root sum of the squares. Decibels add in a more complicated manner. Table 4 provides equations for summing these voltages.

6. Make the appropriate corrections to the power calculations to account for windowing of the FFT, if this has not already been done. If the windowing correction has not been applied previously, it can simply be applied to the final power value calculated. See Equation 9 and Equation 10 for the appropriate correction.

Table to calculate effective voltages over frequency ranges from FFT components.

Step 1. Determine the frequency range you desire.

Step 2. Determine the indices of the FFT bins for this frequency range using the following formula.

$$n(f) = 2 \cdot N \cdot \frac{f}{F_s} = K \cdot \frac{f}{F_s}$$

where: Fs is the sampling rate, K is the number of acquired points used for the FFT, and N is the index in the FFT of the Nyquist frequency. Note that n must be an integer.

Step 3. Determine the voltage units in your FFT.

Step 4. Determine the effective voltage of the frequency range using the equation provided. Note that this effective voltage will be the voltage of a single sinusoid that would deliver the amount of power of the sinusoids in the frequency range.

Units of Voltage	***Effective Voltage Over Frequency Range***
pk-pk	$V_{pp(eff)} = \sqrt{8 \cdot \sum_{n=n_0}^{n_f} \left[\begin{Bmatrix} V_{pp_n} & if & f(n)=0 \\ \frac{V_{pp_n}}{2} & if & f(n)=\frac{F_s}{2} \\ \frac{V_{pp_n}}{2\sqrt{2}} & & otherwise \end{Bmatrix} \right]^2}$ Divide by $2\sqrt{2}$ If the effective pk-pk voltage is being calculated for DC. Divide by $\sqrt{2}$ if the effective pk-pk voltage is being calculated for Nyquist
Amplitude	$A_{eff} = \sqrt{2 \cdot \sum_{n=n_0}^{n_f} \left[\begin{Bmatrix} A_n & if & f(n)=0 \\ A_n & if & f(n)=\frac{F_s}{2} \\ \frac{A_n}{\sqrt{2}} & & otherwise \end{Bmatrix} \right]^2}$ Divide by $\sqrt{2}$ If the effective amplitude is being calculated for either DC or Nyquist
Rms	$V_{rms(eff)} = \sqrt{\sum_{n=n_0}^{n_f} (V_{rms_n})^2}$
DB	$dB_{eff} = 10 \cdot \log\left(\sum_{n=n_0}^{n_f} 10^{\frac{dB_n}{10}} \right)$
DBV	
DBmv	
dBu (or dBv)	
DBW	
DBm	

Table 4 - Table for Effective Voltage Calculations Over Frequency Ranges

Calculating the Figures of Merit

Once the power has been calculated for the components shown in Figure 2, the figures of merit are calculated using the formulas in Table 1.

Averaging

This section is included to provide some insight into further possibilities for improving the measurement results.

As shown in Figure 1, the error portion of the signal is broken into noise and distortion. As mentioned previously, what is classified as noise is usually uncorrelated with the signal and what is classified as distortion usually is. Uncorrelated noise can be removed by averaging. Averaging can only be performed if you are measuring a system capable of generating a stable trigger, like a digital oscilloscope.

You can clearly determine whether certain frequency components of your system are affected by comparing the FFT of the averaged acquisition to the single-shot acquisition. In general, the noise floor will be reduced, and the power of certain distortion components (depending on the source of this distortion) will remain. This enables much easier and accurate calculation of the distortion components.

In the case of uniform and Gaussian (or normal) noise distributions, the noise power decreases by 3 dB for every doubling or by 10 dB for every tenfold increase in the number of acquisitions averaged.

Example

The example provided is for qualitative measurements on a LeCroy WaveMaster 820 Zi-A, a 20 GHz, 40 GS/s Digital Sampling Oscilloscope. The desired measurement is the effective bits due to a 1.499 GHz sinusoid at about +/- 3.5 divisions on the screen. The calculations are performed using a MathCAD 2001i spreadsheet.

This example demonstrates an effective bits calculation of the WaveMaster 820 Zi-A Digital Oscilloscope.

$I := 8$ number of interleaved adcs

$vdiv := 50 \cdot 10^{-3}$ Volts/div setting of the DSO

$F_s := 40$ sampling rate (Gs/s)

$R := 50$ Scope Input Impedence

$bits := 8$ Number of bits in ADC

$freq_{in} := 1.499$ input frequency (GHz)

$K := 20000$ number of points to acquire

$k := 0 .. K - 1$

connection := " "

Read in the points from a file

wfmFileName := "C:\work\WM820ZiA\C1WM820 Zi A_1499.txt"

GetOne(connection) := READPRN(wfmFileName)

x := GetOne(connection)

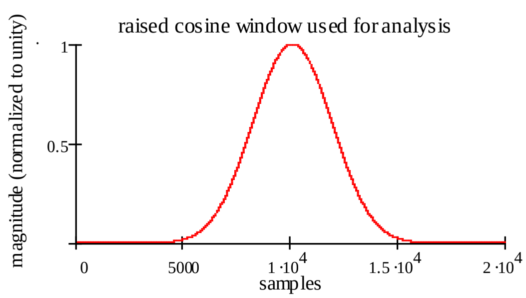
$x_{acq} := x$

Window function used for FFT. This function generates a Kaiser-Bessel window with the specified alpha

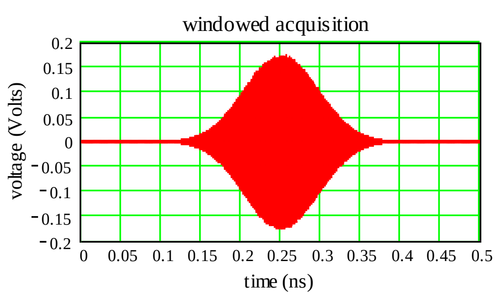
$$window(n, N, \alpha) := \frac{I0\left[\pi \cdot \alpha \cdot \sqrt{1 - \left[\frac{n - \frac{N-1}{2}}{\left(\frac{N}{2}\right)}\right]^2}\right]}{I0(\pi \cdot \alpha)}$$

$w_k := window(k, K, 9.5)$

Use the Kaiser-Bessel window



$xw_k := xacq_k \cdot w_k$ Apply the window



$X := \mathrm{CFFT}(x_w)$ Calculate the DFT

$$n := 0 .. \frac{K}{2} \qquad \mathrm{freq}(n) := \frac{n}{K} \cdot F_s \qquad \mathrm{ni}(f) := \frac{f}{F_s} \cdot K$$

Normalize the DFT to amplitude/phase

$$A_{\frac{K}{2}} := \left| X_{\frac{K}{2}} \right| \qquad A_n := 2 \cdot \left| X_n \right| \qquad A_0 := \left| X_0 \right|$$

calculate the correction (in dBm) to apply to the power due to windowing

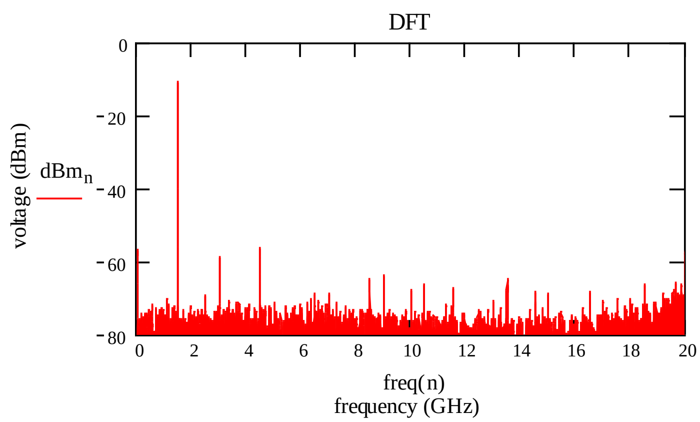
$$\mathrm{corr}_w := -20 \cdot \log\left[\sqrt{\frac{1}{K}\left[\sum_{k=0}^{K-1} \left(w_k\right)^2\right]}\right] \qquad \mathrm{corr}_w = 7.89$$

Convert the DFT to dBm for ease of calculations

$$\mathrm{AtodBm}(A, n, K) := \left| \begin{array}{l} dB \leftarrow 20 \cdot \left(\log\left(\frac{A}{\sqrt{2}}\right)\right) \ldots \\ \quad + -10 \cdot \log(R) \ldots \\ \quad + 30 \\ dB \leftarrow dB + 20 \cdot \log(\sqrt{2}) \ \text{ if } \left[(n = 0) \vee \left(n = \frac{K}{2}\right)\right] \\ \text{return } dB \end{array} \right.$$

$$\mathrm{dBm}_n := \mathrm{AtodBm}\left(A_n, n, K\right) + \mathrm{corr}_w \qquad \max(\mathrm{dBm}) = -10.54$$

Plot of the magnitude spectrum



It is interesting to note that the DFT contains a number of frequency components other than the input signal. These are due to peculiarities in digital oscilloscopes, particular those that interleave multiple ADCs to generate high effective sample rates. When interleaving, the gain/delay/offset of each ADC must match. To the extent that they don't match, they tend to degrade the signal. It is exactly this type of degradation that we are measuring in this example. Keep in mind that in a well designed instrument these components are small (about -47 dB) and do not generally affect normal measurements

Now that the DFT has been taken, the next step is to classify all of the frequencies of interest. These will include the input signal frequency, the DC point, and the noise and distortion components.

classification of frequency components:

Harmonics

$H := 5 \qquad h := 0..H-1$

$f_{harm_h} := (h+1)\cdot freq_{in} \qquad des_{harm_h} := h+1$

$$f_{harm_h} := \mathrm{if}\left[f_{harm_h} > \frac{F_S}{2}, \frac{F_S}{2} - \left(f_{harm_h} - \frac{F_S}{2}\right), f_{harm_h}\right]$$

.

$$f_{harm_h} := \mathrm{if}\left(f_{harm_h} < 0, -f_{harm_h}, f_{harm_h}\right)$$

$$f_{harm_h} := \mathrm{if}\left[f_{harm_h} > \frac{F_S}{2}, \frac{F_S}{2} - \left(f_{harm_h} - \frac{F_S}{2}\right), f_{harm_h}\right]$$

$$des_{harm_h} = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{pmatrix} \qquad f_{harm_h} = \begin{pmatrix} 1.5 \\ 3 \\ 4.5 \\ 6 \\ 7.5 \end{pmatrix}$$

Note: These equations 'fold' harmonics that are above the Nyquist frequency back into the baseband spectrum.

adc offset mismatch components

$O := \mathrm{floor}\left(\frac{I}{2} + 1\right) \qquad o := 0..O-1$

$f_{offset_o} := \frac{F_S}{I}\cdot o \qquad des_{offset_o} := H + o + 1 \qquad o = \begin{pmatrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \end{pmatrix}$

$O = 5$

$$f_{offset_o} = \begin{pmatrix} 0 \\ 5 \\ 10 \\ 15 \\ 20 \end{pmatrix} \qquad des_{offset_o} = \begin{pmatrix} 6 \\ 7 \\ 8 \\ 9 \\ 10 \end{pmatrix}$$

Note that offset mismatch frequencies are functions of the sampling rate and independent of the signal frequency.

adc gain/delay mismatch components

$$G := I \qquad g := 0..G-1$$

$$f_{gdf_h} := \left[\frac{f_{harm_h}}{\left(\frac{F_s}{I}\right)} - \mathrm{floor}\left[\frac{f_{harm_h}}{\left(\frac{F_s}{I}\right)}\right]\right]\cdot\frac{F_s}{I}$$

$$f_{gdf_h} = \begin{pmatrix} 1.5 \\ 3 \\ 4.5 \\ 1 \\ 2.5 \end{pmatrix}$$

$$f_{gd_{g,h}} := \frac{F_s \cdot \mathrm{floor}\left(\frac{g+1}{2}\right)}{I} + (-1)^g \cdot f_{gdf_h}$$

$$f_{gd_{g,h}} := \mathrm{if}\left(f_{gd_{g,h}} < 0, -f_{gd_{g,h}}, f_{gd_{g,h}}\right)$$

$$f_{gd_{g,h}} := \mathrm{if}\left(f_{gd_{g,h}} > \frac{F_s}{2}, F_s - f_{gd_{g,h}}, f_{gd_{g,h}}\right)$$

$$f_{gd_{g,h}} := \mathrm{if}\left(f_{gd_{g,h}} = f_{harm_h}, -1, f_{gd_{g,h}}\right)$$

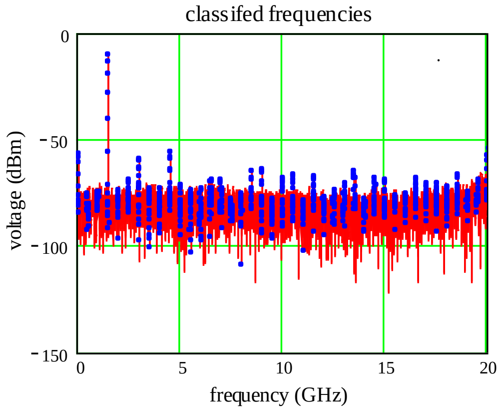
$$des_{gd_{g,h}} := O + H + h \cdot G + g + 1$$

$$f_{gd} = \begin{pmatrix} -1 & -1 & -1 & 1 & 2.5 \\ 3.5 & 2 & 0.5 & 4 & 2.5 \\ 6.5 & 8 & 9.5 & -1 & -1 \\ 8.5 & 7 & 5.5 & 9 & 7.5 \\ 11.5 & 13 & 14.5 & 11 & 12.5 \\ 13.5 & 12 & 10.5 & 14 & 12.5 \\ 16.5 & 18 & 19.5 & 16 & 17.5 \\ 18.5 & 17 & 15.5 & 19 & 17.5 \end{pmatrix}$$

Note that some elements contain a frequency of -1. This is intentional since these components are images of the input signal and its harmonics. The negative frequency will cause them to be unused in the

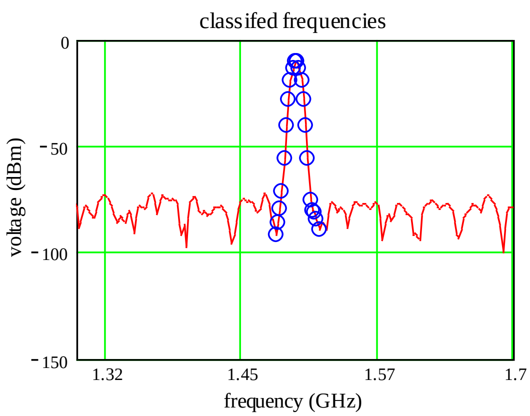
At this point, it is interesting to take a look at how the designated frequencies line up with the peaks in the DFT

Here is a plot of the DFT with the classified components overlaid



The designated frequencies are highlighted in blue. The designated frequencies seem to cover most of the peaks, but it seems that it covers the entire spectrum. This would not be desirable because there has to be some room in between the spectral components to be designated as noise.

Here is a zoom of the input frequency:
From this picture, it is clear that the selection of 20 MHz as the delta frequency is fine. Note that you do not need to get the delta frequency exactly correct. You must only ensure that the designation "coats" the peak down to the noise floor. It does not matter if it coats some of the noise - this is accounted for later in the calculations.



$$\mathrm{MedianPower}(n_s, n_e, D, dBm) := \left| \begin{array}{l} n_s \leftarrow 0 \ \ \text{if} \ \ n_s < 0 \\ n_e \leftarrow \mathrm{last}(D) - 1 \ \ \text{if} \ \ n_e > \mathrm{last}(D) - 1 \\ m \leftarrow 0 \\ da_m \leftarrow -3000 \\ \text{for} \ \ n \in n_s .. n_e \\ \quad \text{if} \ \ D_n = 0 \\ \qquad \left| \begin{array}{l} da_m \leftarrow dBm_n \\ m \leftarrow m + 1 \end{array} \right. \\ \mathrm{median}(da) \end{array} \right.$$

Let's plot the designated noise

$$grain := .5$$

$$S := \frac{freq\left(\frac{K}{2}\right) - freq(0)}{grain}$$

$$S = 40$$

$$s := 0..S$$

$$fi_s := s \cdot grain$$

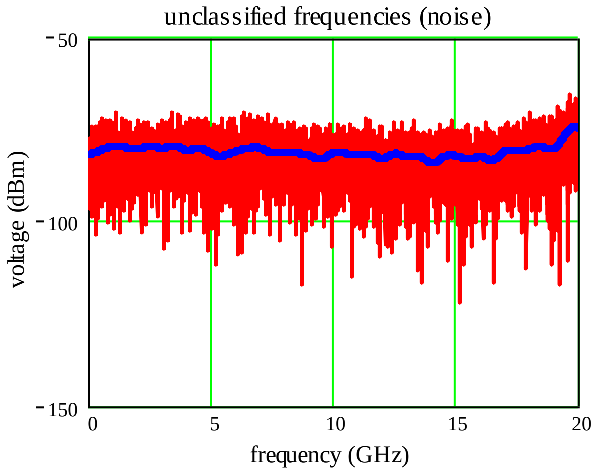
$$dBm_{median_s} := MedianPower\left(floor\left(ni\left(fi_s - \frac{grain}{2}\right)\right), ceil\left(ni\left(fi_s + \frac{grain}{2}\right)\right), D, dBm\right)$$

$$dBms_{median} := cspline\left(fi, dBm_{median}\right)$$

$$MedianNoiseFloor_n := interp\left(dBms_{median}, fi, dBm_{median}, freq(n)\right)$$

$$dBmf_n := if\left(D_n, MedianNoiseFloor_n, dBm_n\right)$$

Here is a plot of the undesignated components
(i.e. designated as noise):



You can see that except for a few spurious components, all of the frequencies have been classified and the noise is a good representation of a noise floor

Noise calculation:
Noise is calculated by summing the effective voltage of the undesignated DFT components, then calculating an effective noise per DFT bin, then summing them as if it were present in every bin (which it is).

$$F_{Nyquist} := \frac{F_s}{2} \qquad F_{Nyquist} = 20$$

$$F_{pbe} := 1.5 \qquad F_{sbe} := \frac{F_s \cdot .8}{2} \qquad F_{sbe} = 16$$

$$P_n := 10 \cdot \log\left(\sum_{n = ni(0)}^{ni(F_{Nyquist})} 10^{\frac{dBmf_n}{10}} \right)$$

$$P_n = -39.23$$

Now, we go ahead and calculate all of the powers of the designated components and add their powers to further group them (for example, the power of all of the harmonics are added to form the harmonic distortion power, etc.). The tabulated results of all of these calculations are provided at the end.

calculate the powers (effective voltages) of each classification

$$\begin{aligned}
\text{Power} := \;& \text{for } p \in 0..100 \\
& \quad P_p \leftarrow 10^{\left(\frac{-3000}{10}\right)} \\
& \text{for } n \in 0..\frac{K}{2} \\
& \quad P_{(D_n)} \leftarrow P_{(D_n)} + \left(10^{\frac{dBm_n}{10}} - 10^{\frac{dBmf_n}{10}} \right) \\
& \text{for } p \in 0..100 \\
& \quad \left| \begin{array}{l} P_p \leftarrow 10 \cdot \log(P_p) \\ P_p \leftarrow -3000 \;\; \text{if } \; \mathrm{Im}(P_p) \neq 0 \end{array} \right. \\
& P
\end{aligned}$$

Computing the power in the harmonics

$$P_{harm_h} := Power\left(des_{harm_h}\right)$$

$$P_{harm_h} = \begin{pmatrix} -5.26 \\ -53.34 \\ -50.8 \\ -71.53 \\ -85.24 \end{pmatrix} \qquad f_{harm} = \begin{pmatrix} 1.5 \\ 3 \\ 4.5 \\ 6 \\ 7.5 \end{pmatrix}$$

Power in the harmonics

$$P_h := 10 \cdot \log\left(\sum_{i=1}^{H-1} 10^{\frac{P_{harm_i}}{10}}\right)$$

$$P_h = -48.85$$

Power in the fundamental

$$P_f := P_{harm_0} \qquad P_f = -5.26$$

Calculating the power in the offset mismatch

$$P_{offset_o} := Power\left(des_{offset_o}\right)$$

$$P_{offset_o} = \begin{pmatrix} -53.18 \\ -67.17 \\ -62.51 \\ -64.15 \\ -51.57 \end{pmatrix}$$

$$P_{om} := 10 \cdot \log\left(\sum_{i=1}^{O-1} 10^{\frac{P_{offset_i}}{10}}\right) \qquad P_{om} = -50.91$$

$$P_o := P_{offset_0} \qquad P_o = -53.18$$

Determination of the power due to gain/delay mismatch

$$P_{gd_{g,h}} := \text{Power}\left(des_{gd_{g,h}}\right)$$

$$des_{gd} = \begin{pmatrix} 11 & 19 & 27 & 35 & 43 \\ 12 & 20 & 28 & 36 & 44 \\ 13 & 21 & 29 & 37 & 45 \\ 14 & 22 & 30 & 38 & 46 \\ 15 & 23 & 31 & 39 & 47 \\ 16 & 24 & 32 & 40 & 48 \\ 17 & 25 & 33 & 41 & 49 \\ 18 & 26 & 34 & 42 & 50 \end{pmatrix} \qquad f_{gd} = \begin{pmatrix} -1 & -1 & -1 & 1 & 2.5 \\ 3.5 & 2 & 0.5 & 4 & 2.5 \\ 6.5 & 8 & 9.5 & -1 & -1 \\ 8.5 & 7 & 5.5 & 9 & 7.5 \\ 11.5 & 13 & 14.5 & 11 & 12.5 \\ 13.5 & 12 & 10.5 & 14 & 12.5 \\ 16.5 & 18 & 19.5 & 16 & 17.5 \\ 18.5 & 17 & 15.5 & 19 & 17.5 \end{pmatrix}$$

$$P_{gd} = \begin{pmatrix} -3\times10^3 & -3\times10^3 & -3\times10^3 & -70.21 & -75.33 \\ -68.21 & -68.06 & -76.79 & -69.32 & -63.38 \\ -62.95 & -71.3 & -86.26 & -3\times10^3 & -3\times10^3 \\ -59.07 & -63.79 & -70.9 & -59.01 & -3\times10^3 \\ -61.61 & -65.24 & -61.26 & -3\times10^3 & -78.21 \\ -56.95 & -71.64 & -61.62 & -71.49 & -66.85 \\ -60.87 & -66.34 & -68.71 & -66.86 & -70.84 \\ -61.27 & -64.33 & -68.38 & -70.75 & -64.66 \end{pmatrix}$$

$$P_{gdm_h} := 10\cdot\log\left(\sum_{g=0}^{G-1} 10^{\frac{P_{gd_{g,h}}}{10}}\right)$$

$$P_{gdm} := 10\cdot\log\left(\sum_{h=0}^{H-1} 10^{\frac{P_{gdm_h}}{10}}\right) \qquad P_{gdm} = -49.01$$

Summing the power due to distrotion components

$$P_d := 10\cdot\log\left(10^{\frac{P_h}{10}} + 10^{\frac{P_{gdm}}{10}} + 10^{\frac{P_{om}}{10}}\right) \qquad P_d = -44.72$$

Computing the power due to noise and disitortion

$$P_{nad} := 10\cdot\log\left(10^{\frac{P_d}{10}} + 10^{\frac{P_n}{10}}\right) \qquad P_{nad} = -38.15$$

Computing the power due to interleaving components

$$P_{id} := 10\cdot\log\left(10^{\frac{P_{om}}{10}} + 10^{\frac{P_{gdm}}{10}}\right) \qquad P_{id} = -46.84$$

$$SNR := P_f - P_n \qquad SNR = 33.96 \qquad P_f = -5.26$$

$$SDR := P_f - P_d \qquad SDR = 39.46$$

$$SINAD := P_f - P_{nad} \qquad SINAD = 32.88$$

$$ENOB := \frac{SINAD - 10\cdot\log\left(\frac{3}{2}\right)}{20\cdot\log(2)} \qquad ENOB = 5.17$$

$$P_{fs} := 20\cdot\log\left(\frac{4\cdot vdiv}{\sqrt{2}}\right) - 10\cdot\log(R) + 30 \qquad P_{fs} = -3.98$$

$$P_{fs} - P_f = 1.28$$

$$\Delta P := P_{fs} - P_f$$

$$Adj := \frac{\Delta P - 10\cdot\log\left[10^{\frac{-SINAD}{10}}\cdot 10^{\frac{\Delta P}{10}} + 10^{\frac{-SNR}{10}}\cdot\left(1 - 10^{\frac{\Delta P}{10}}\right)\right] - 1.76}{6.02} - ENOB$$

$$Adj = 0.16 \qquad \frac{P_{fs} - P_f}{6.02} = 0.21$$

$$ENOB_{adjusted} := ENOB + Adj$$

$$ENOB_{adjusted} = 5.33$$

Name	*Frequency (GHz)*	*Effective Voltage (dBm)*
Fundamental (P_f)	1.499	-5.26
Full-scale Input (P_{fs})	calculated	-3.98
Noise and Distortion (P_{nad})	Full Bandwidth	-38.15
Noise		-39.23
Distortion (P_d)		-44.72
Harmonics (P_h)		-48.85
ADC Offset mismatch components (P_{om})		-53.18
ADC Gain/Delay mismatch components (P_{gdm})		-49.01

Figure of Merit	**Definition**	**Value**
SINAD (dB)	$P_f - P_{nad}$	32.88
ENOB	$\frac{SINAD - 1.76}{6.02}$	5.17
$ENOB_{adj}$	$\mathrm{ENOB}_{\mathrm{adjusted}} := \mathrm{ENOB} + \mathrm{Adj}$	5.33
SNR	Pfs - N0	33.96

Example Summary

An example was shown of an effective bits measurement on a LeCroy DSO. The example was illustrative of many items not shown in the paper and is indicative of the cleverness which can be employed once the power measurement techniques are mastered.

In particular, it illustrated:

1. The normalization, conversion to dBm and correction for windowing.
2. Methods of classification of spectral components using MathCAD.
3. The result of proper classification in the lack of peaks in the DFT of the remaining noise.
4. The proper determination of spectral component size (delta f).
5. The distribution of noise power over other designated components.
6. The determination of quantization and Gaussian noise, and the ability to analyze the actual number of bits in the digitizer.
7. The estimation of the noise floor function.
8. The calculation of powers of components by subtracting off the underlying noise.
9. The further determination of the power of classes of components such has harmonic power, offset mismatch power, etc.
10. The ability to define figures of merit unique to a particular measurement.

Tables

Table to convert between various voltage units.

In order to use this table, you must first know the following values:
R, the assumed resistance which the voltage appears across and P_{ref}, the reference power level.
Then, simply cross-reference the units of your input and output variable to find the correct equation. Then plug in the numbers.

Example: You are working on RF and want to find the rms voltage corresponding to -10 dBm.

Step 1. Determine R. For RF, R is assumed to be 50 Ω.
Step 2. Determine P_{ref}. For RF, P_{ref} is assumed to be 1 mW.
Step 3. Find the column containing dBm.
Step 4. Look for the row containing rms.

The equation is: $v_{rms} = 10^{\frac{\left(dBm + 10\cdot\log(R) - 30\right)}{20}}$

Step 5. Plug in the numbers – R=50, dBm = -10. -10 dBm = 70.711 mV (rms).

	From								
To	pk-pk	Amplitude	rms	dB	dBV	dBmv	DBu (or dBv)	dBW	dBm
pk-pk	---	$V_{pp} = A\cdot 2$ except DC where $V_{pp} = A$	$V_{pp} = V_{rms}\cdot 2\sqrt{2}$ except DC where $V_{pp} = V_{rms}$ or Nyquist where $V_{pp} = V_{rms}\cdot 2$	$V_{pp} = 2\sqrt{2}\cdot 10^{\frac{\left(dB + 10\cdot\log(R) + 10\cdot\log(P_{ref})\right)}{20}}$ divide by $2\sqrt{2}$ for DC divide by $\sqrt{2}$ for Nyquist	$V_{pp} = 2\sqrt{2}\cdot 10^{\frac{dBV}{20}}$ divide by $2\sqrt{2}$ for DC divide by $\sqrt{2}$ for Nyquist	$V_{pp} = 2\sqrt{2}\cdot 10^{\frac{(dBmv-60)}{20}}$ divide by $2\sqrt{2}$ for DC divide by $\sqrt{2}$ for Nyquist	$V_{pp} = 2\sqrt{2}\cdot 10^{\frac{(dBu-120)}{20}}$ divide by $2\sqrt{2}$ for DC divide by $\sqrt{2}$ for Nyquist	$V_{pp} = 2\sqrt{2}\cdot 10^{\frac{\left(dB + 10\cdot\log(R)\right)}{20}}$ divide by $2\sqrt{2}$ for DC divide by $\sqrt{2}$ for Nyquist	$V_{pp} = 2\sqrt{2}\cdot 10^{\frac{\left(dBm + 10\cdot\log(R) - 30\right)}{20}}$ divide by $2\sqrt{2}$ for DC divide by $\sqrt{2}$ for Nyquist
Amplitude	$A = \frac{V_{pp}}{2}$ except DC where $A = V_{pp}$	---	$A = V_{rms}\cdot\sqrt{2}$ except DC and Nyquist where $A = V_{rms}$	$A = \sqrt{2}\cdot 10^{\frac{\left(dB + 10\cdot\log(R) + 10\cdot\log(P_{ref})\right)}{20}}$ divide by $\sqrt{2}$ for DC and Nyquist	$A = \sqrt{2}\cdot 10^{\frac{dBV}{20}}$ divide by $\sqrt{2}$ for DC and Nyquist	$A = \sqrt{2}\cdot 10^{\frac{(dBmv-60)}{20}}$ divide by $\sqrt{2}$ for DC and Nyquist	$A = \sqrt{2}\cdot 10^{\frac{(dBu-120)}{20}}$ divide by $\sqrt{2}$ for DC and Nyquist	$A = \sqrt{2}\cdot 10^{\frac{\left(dB + 10\cdot\log(R)\right)}{20}}$ divide by $\sqrt{2}$ for DC and Nyquist	$A = \sqrt{2}\cdot 10^{\frac{\left(dBm + 10\cdot\log(R) - 30\right)}{20}}$ divide by $\sqrt{2}$ for DC and Nyquist
rms	$V_{rms} = \frac{V_{pp}}{2\sqrt{2}}$ except DC where $V_{rms} = V_{pp}$ or Nyquist where $V_{rms} = \frac{V_{pp}}{2}$	$V_{rms} = \frac{A}{\sqrt{2}}$ except DC and Nyquist where $V_{rms} = A$	---	$v_{rms} = 10^{\frac{\left(dB + 10\cdot\log(R) + 10\cdot\log(P_{ref})\right)}{20}}$	$v_{rms} = 10^{\frac{dBV}{20}}$	$v_{rms} = 10^{\frac{(dBmv-60)}{20}}$	$v_{rms} = 10^{\frac{(dBu-120)}{20}}$	$v_{rms} = 10^{\frac{\left(dB + 10\cdot\log(R)\right)}{20}}$	$v_{rms} = 10^{\frac{\left(dBm + 10\cdot\log(R) - 30\right)}{20}}$
dB	$dB = 20\cdot\log\left(\frac{V_{pp}}{2\sqrt{2}}\right)$ $-10\cdot\log(R)$ $-10\cdot\log(P_{ref})$ add 9.031 for DC add 3.01 for Nyquist	$dB = 20\cdot\log\left(\frac{A}{\sqrt{2}}\right)$ $-10\cdot\log(R)$ $-10\cdot\log(P_{ref})$ add 3.01 for DC and Nyquist	$dB =$ $20\cdot\log(v_{rms})$ $-10\cdot\log(R)$ $-10\cdot\log(P_{ref})$	---	$dB =$ dBV $-10\cdot\log(R)$ $-10\cdot\log(P_{ref})$	$dB =$ $dBmv$ $-10\cdot\log(R)$ $-10\cdot\log(P_{ref})$ -60	$dB =$ dBu $-10\cdot\log(R)$ $-10\cdot\log(P_{ref})$ -120	$dB =$ dBW $-10\cdot\log(P_{ref})$	$dB =$ dBm $-10\cdot\log(P_{ref})$ -30
dBV	$dBV = 20\cdot\log\left(\frac{V_{pp}}{2\sqrt{2}}\right)$ add 9.031 for DC add 3.01 for Nyquist	$dBV = 20\cdot\log\left(\frac{A}{\sqrt{2}}\right)$ add 3.01 for DC and Nyquist	$dBV =$ $20\cdot\log(v_{rms})$	$dBV =$ dB $+10\cdot\log(R)$ $+10\cdot\log(P_{ref})$	---	$dBV =$ $dBmv - 60$	$dBV =$ $dBu - 120$	$dBV =$ dBW $+10\cdot\log(R)$	$dBV =$ dBm $+10\cdot\log(R)$ -30
dBmv	$dBmv = 20\cdot\log\left(\frac{V_{pp}}{2\sqrt{2}}\right)$ $+60$ add 9.031 for DC add 3.01 for Nyquist	$dBmv = 20\cdot\log\left(\frac{A}{\sqrt{2}}\right)$ $+60$ add 3.01 for DC and Nyquist	$dBmv =$ $20\cdot\log(v_{rms})$ $+60$	$dBmv =$ dB $+10\cdot\log(R)$ $+10\cdot\log(P_{ref})$ $+60$	$dBmv =$ $dBV + 60$	---	$dBmv =$ $dBu - 60$	$dBmv =$ dBW $+10\cdot\log(R)$ $+60$	$dBmv =$ dBm $+10\cdot\log(R)$ $+30$
dBu (or dBv)	$dBu = 20\cdot\log\left(\frac{V_{pp}}{2\sqrt{2}}\right)$ $+120$ add 9.031 for DC add 3.01 for Nyquist	$dBu = 20\cdot\log\left(\frac{A}{\sqrt{2}}\right)$ $+120$ add 3.01 for DC and Nyquist	$dBu =$ $20\cdot\log(v_{rms})$ $+120$	$dBu =$ dB $+10\cdot\log(R)$ $+10\cdot\log(P_{ref})$ $+120$	$dBu =$ $dBV + 120$	$dBu =$ $dBV + 60$	---	$dBu =$ dBW $+10\cdot\log(R)$ $+120$	$dBu =$ dBm $+10\cdot\log(R)$ $+90$
dBW	$dBW = 20\cdot\log\left(\frac{V_{pp}}{2\sqrt{2}}\right)$ $-10\cdot\log(R)$ add 9.031 for DC add 3.01 for Nyquist	$dBW = 20\cdot\log\left(\frac{A}{\sqrt{2}}\right)$ $-10\cdot\log(R)$ add 3.01 for DC and Nyquist	$dBW =$ $20\cdot\log(v_{rms})$ $-10\cdot\log(R)$	$dBW =$ dB $+10\cdot\log(P_{ref})$	$dBW =$ dBV $-10\cdot\log(R)$	$dBW =$ $dBmv$ $-10\cdot\log(R)$ -60	$dBW =$ dBu $-10\cdot\log(R)$ -120	---	$dBW =$ dBm -30
dBm	$dBm = 20\cdot\log\left(\frac{V_{pp}}{2\sqrt{2}}\right)$ $-10\cdot\log(R)$ $+30$ add 9.031 for DC add 3.01 for Nyquist	$dBm = 20\cdot\log\left(\frac{A}{\sqrt{2}}\right)$ $-10\cdot\log(R)$ $+30$ add 3.01 for DC and Nyquist	$dBm =$ $20\cdot\log(v_{rms})$ $-10\cdot\log(R)$ $+30$	$dBm =$ dB $+10\cdot\log(P_{ref})$ $+30$	$dBm =$ dBV $-10\cdot\log(R)$ $+30$	$dBm =$ $dBmv$ $-10\cdot\log(R)$ -30	$dBm =$ dBu $-10\cdot\log(R)$ -90	$dBm =$ dBW $+30$	---

Table 5 - Table for Voltage Unit Conversion

Table to convert between various voltage units – tailored to RF

R, the assumed resistance that the voltage appears across is assumed to be 50 Ω.
P_{ref}, the reference power level, is assumed to be 1 mW

Simply select the column containing the units of your input variable and look down the column for the row containing the units you are converting to. Then plug in the numbers.

Example: Find the rms voltage corresponding to -10 dBm.

Step 1. Find the column containing dBm.
Step 2. Look for the row containing rms.

The equation is: $v_{rms} = 10^{\frac{dBm}{20}} \cdot 223.607 \cdot 10^{-3}$

Step 5. Plug in dBm = -10. -10 dBm = 70.711 mV (rms).

	From								
To	pk-pk	Amplitude	rms	dB	dBV	dBmv	DBu (or dBv)	dBW	dBm
pk-pk	---	$V_{pp} = A \cdot 2.000$ except DC where $V_{pp} = A$	$V_{pp} = V_{rms} \cdot 2.828$ except DC where $V_{pp} = V_{rms}$ or Nyquist where $V_{pp} = V_{rms} \cdot 2.000$	$V_{pp} = 10^{\frac{dB}{20}} \cdot 632.454 \cdot 10^{-3}$ divide by 2.828 for DC divide by 1.414 for Nyquist	$V_{pp} = 10^{\frac{dBV}{20}} \cdot 2.828$ divide by 2.828 for DC divide by 1.414 for Nyquist	$V_{pp} = 10^{\frac{dBmv}{20}} \cdot 2.828 \cdot 10^{-3}$ divide by 2.828 for DC divide by 1.414 for Nyquist	$V_{pp} = 10^{\frac{dBu}{20}} \cdot 2.828 \cdot 10^{-6}$ divide by 2.828 for DC divide by 1.414 for Nyquist	$V_{pp} = 10^{\frac{dB}{20}} \cdot 20.000$ divide by 2.828 for DC divide by 1.414 for Nyquist	$V_{pp} = 10^{\frac{dBm}{20}} \cdot 632.456$ divide by 2.828 for DC divide by 1.414 for Nyquist
Amplitude	$A = V_{pp} \cdot 0.500$ except DC where $A = V_{pp}$	---	$A = V_{rms} \cdot 1.414$ except DC and Nyquist where $A = V_{rms}$	$A = 10^{\frac{dB}{20}} \cdot 316.228 \cdot 10^{-3}$ divide by 1.414 for DC and Nyquist	$A = 10^{\frac{dBV}{20}} \cdot 1.414$ divide by 1.414 for DC and Nyquist	$A = 10^{\frac{dBmv}{20}} \cdot 1.414 \cdot 10^{-3}$ divide by 1.414 for DC and Nyquist	$A = 10^{\frac{dBu}{20}} \cdot 1.414 \cdot 10^{-6}$ divide by 1.414 for DC and Nyquist	$A = 10^{\frac{dB}{20}} \cdot 10.000$ divide by 1.414 for DC and Nyquist	$A = 10^{\frac{dBm}{20}} \cdot 316.228 \cdot 10^{-3}$ divide by 1.414 for DC and Nyquist
rms	$V_{rms} = V_{pp} \cdot 353.553 \cdot 10^{-3}$ except DC where $V_{rms} = V_{pp}$ or Nyquist where $V_{rms} = V_{pp} \cdot 0.500 \cdot 10^{-3}$	$V_{rms} = A \cdot 707.107 \cdot 10^{-3}$ except DC and Nyquist where $V_{rms} = A$	---	$v_{rms} = 10^{\frac{dB}{20}} \cdot 223.607 \cdot 10^{-3}$	$v_{rms} = 10^{\frac{dBV}{20}}$	$v_{rms} = 10^{\frac{dBmv}{20}} \cdot 1.000 \cdot 10^{-3}$	$v_{rms} = 10^{\frac{dBu}{20}} \cdot 1.000 \cdot 10^{-6}$	$v_{rms} = 10^{\frac{dB}{20}} \cdot 7.071$	$v_{rms} = 10^{\frac{dBm}{20}} \cdot 223.607 \cdot 10^{-3}$
dB	$dB = 20 \cdot \log(V_{pp}) + 3.979$ add 9.031 for DC add 3.01 for Nyquist	$dB = 20 \cdot \log(A) + 10.000$ add 3.01 for DC and Nyquist	$dB = 20 \cdot \log(v_{rms}) + 13.010$	---	$dB = dBV + 13.010$	$dB = dBmv - 46.990$	$dB = dBu - 106.990$	$dB = dBW + 30.000$	$dB = dBm$
dBV	$dBV = 20 \cdot \log(V_{pp}) - 9.031$ add 9.031 for DC add 3.01 for Nyquist	$dBV = 20 \cdot \log(A) - 3.01$ add 3.01 for DC and Nyquist	$dBV = 20 \cdot \log(v_{rms})$	$dBV = dB - 13.010$	---	$dBV = dBmv - 60.000$	$dBV = dBu - 120.000$	$dBV = dBW + 16.990$	$dBV = dBm - 13.010$
dBmv	$dBmv = 20 \cdot \log(V_{pp}) + 50.969$ add 9.031 for DC add 3.01 for Nyquist	$dBmv = 20 \cdot \log(A) + 56.990$ add 3.01 for DC and Nyquist	$dBmv = 20 \cdot \log(v_{rms}) + 60.000$	$dBmv = dB + 46.990$	$dBmv = dBV + 60.000$	---	$dBmv = dBu - 60.000$	$dBmv = dBW + 76.990$	$dBmv = dBm + 46.990$
dBu (or dBv)	$dBu = 20 \cdot \log(V_{pp}) + 110.969$ add 9.031 for DC add 3.01 for Nyquist	$dBu = 20 \cdot \log(A) + 116.990$ add 3.01 for DC and Nyquist	$dBu = 20 \cdot \log(v_{rms}) + 120.000$	$dBu = dB + 106.990$	$dBu = dBV + 120.000$	$dBu = dBV + 60.000$	---	$dBu = dBW + 136.990$	$dBu = dBm + 106.990$
dBW	$dBW = 20 \cdot \log(V_{pp}) - 26.021$ add 9.031 for DC add 3.01 for Nyquist	$dBW = 20 \cdot \log(A) - 20.000$ add 3.01 for DC and Nyquist	$dBW = 20 \cdot \log(v_{rms}) - 16.990$	$dBW = dB - 30.000$	$dBW = dBV - 16.990$	$dBW = dBmv - 76.990$	$dBW = dBu - 136.990$	---	$dBW = dBm - 30.000$
dBm	$dBm = 20 \cdot \log(V_{pp}) + 3.979$ add 9.031 for DC add 3.01 for Nyquist	$dBm = 20 \cdot \log(A) + 10.000$ add 3.01 for DC and Nyquist	$dBm = 20 \cdot \log(v_{rms}) + 13.010$	$dBm = dB$	$dBm = dBV + 13.010$	$dBm = dBmv - 46.990$	$dBm = dBu - 106.990$	$dBm = dBW + 30.000$	---

Table 6 - Table for Voltage Unit Conversion (RF measurements - R=50Ω, Pref=1 mW)

Background Information

RMS Voltage of Quantization Noise

The rms voltage is the square root of the sum of the square of each voltage sample. In the case of the rms value of noise, it is the square root of the sum of the square of the difference between the voltage sample and the ideal voltage value. For example, if the ideal voltage in a sequence of K samples can be expressed as x_k, and the actual voltage samples are y_k, the rms value of the noise is calculated as:

$$V_{rms} = \int_{-\frac{1}{2}}^{\frac{1}{2}} x^2 \cdot dx = \frac{1}{2\sqrt{3}}$$

Equation 15

For gaussian noise sources, a histogram of all of the values of ε_k will resemble, as the name implies, a gaussian distribution. Equation is the same as the equation of standard deviation.

The histogram of the values of ε_k is a uniform distribution in the case of quantization noise. This can be understood conceptually by first understanding that the maximum error on each sample is at most 1/2 of a code, and the error has an equal probability of lying between +/- 1/2 code.

The rms value of this uniform distribution (in codes) can be expressed as the following integral:

$$\varepsilon_k = (y_k - x_k)$$

$$V_{rms} = \sqrt{\frac{1}{K} \cdot \sum_{k=0}^{K-1} \varepsilon_k{}^2}$$

Equation 16 - rms voltage of quantization noise (in codes)

Derivation of ENOB Equation

The definition of the effective number of bits is the number of bits in an ideal quantizer in a system with no other noise sources (excepting quantization noise) that has the equivalent amount of noise to the system being measured. The calculation of ENOB can be derived by considering how much noise such a system would have.

First of all, we know from the section "**Error! Reference source not found.**", that quantization creates an rms error (or noise voltage) of $\frac{1}{2\sqrt{3}}$ codes. Assuming a b bit quantizer, the maximum peak-peak voltage of a sinusoid quantized by this system is 2^b, which is a sinusoid with an amplitude and rms value of 2^{b-1} and $\frac{2^{b-1}}{\sqrt{2}}$, respectively. Thus, the signal to noise ratio of such a system can be written as:

$$snr = 20 \cdot \log\left(\frac{\left(\frac{2^{b-1}}{\sqrt{2}}\right)}{\left(\frac{1}{2\sqrt{3}}\right)}\right)$$ which can be simplified as

$$snr = 20 \cdot \log(2^b) + 20 \cdot \log\left(\frac{2^{-1} \cdot 2 \cdot \sqrt{3}}{\sqrt{2}}\right)$$

and further as

$$snr = b \cdot 20 \cdot \log(2) + 10 \cdot \log\left(\frac{3}{2}\right)$$

Solving for b yields:

$$b = \frac{snr - 10 \cdot \log\left(\frac{3}{2}\right)}{20 \cdot \log(2)}$$

or $$b = \frac{snr - 1.761}{6.021}$$

Equation 17

Thus, ENOB and SNR (or more exactly, SINAD) are equivalent measurement parameters.

Derivation of ENOB Adjustment

Given an ENOB measurement that was not taken with a full-scale sine wave, we show here how to make the correct adjustment for ENOB were the measurement actually taken at full-scale. To start, we are given the following parameters:

- P_f - the power of the fundamental – the sine wave used for the measurement.
- P_{fs} - the power of a full-scale sine wave.
- $SINAD$ - the signal-to-noise-and-distortion measurement taken with the sine wave not at full-scale.
- SNR - the signal-to-noise measurement taken with the sine wave not at full-scale.

- The power of the noise in the actual measurement is:

$$P_n = P_f - SNR$$

[1]

and the power of the noise and distortion in the actual measurement is:

$$P_{nad} = P_f - SINAD$$

[2]

The signal-to-distortion ratio is the uncorrelated power subtraction expressed by:

$$SDR = -10 \cdot \mathrm{Log}_{10}\left(10^{-\frac{SINAD}{10}} - 10^{-\frac{SNR}{10}}\right)$$

[3]

The power of the distortion components in the actual measurement is therefore:

$$P_d = P_f - SDR$$

[4]

The premise here is that when making the adjustment for full-scale, the actual sinewave is adjusted by the amount $\Delta P = P_{fs} - P_f$, which causes the power of the distortion components to rise by the same ΔP, but causes no rise in the power of the noise. Therefore, the value of the new fundamental is: $P_f' = P_f + \Delta P = P_{fs}$, the new value of the distortion is $P_d' = P_d + \Delta P$ and the new value of the noise is unchanged as: $P_n' = P_n$. Therefore the new signal-to-noise measurement is:

$$SNR' = P_f' - P_n' = P_f + \Delta P - P_n = SNR + \Delta P$$

[5]

The new signal-to-distortion ratio is:

$$SDR' = P_f' - P_d' = P_f + \Delta P - P_d - \Delta P = SDR\ .$$

Therefore, the new signal-to-noise-and-distortion ratio is:

$$\Delta SINAD = SINAD' - SINAD = -10 \cdot \mathrm{Log}_{10}\left(10^{-\frac{SINAD}{10}} + 10^{-\frac{SNR}{10}}\left(10^{-\frac{\Delta P}{10}} - 1\right)\right) - SINAD$$

[6]

Since effective-number-of-bits is calculated as:

$$ENOB = \frac{SINAD - 1.76}{6.02}$$

The new effective-number-of-bits is calculated as:

$$ENOB' = \frac{SINAD' - 1.76}{6.02}$$

And the ENOB correction is given by:

$$\Delta ENOB = ENOB' - ENOB = \frac{\Delta SINAD}{6.02}$$

Let's make some simple checks. Let's assume that all of the degradation is due to noise which can be the case only if there is some noise, but no distortion. This means that the power of the distortion is $-\infty$, which means according to [4] that $SDR = \infty$ and according to [3] that $SNR = SINAD$.

Therefore, according to [6], the adjustment for a full-scale sinewave is:

$$\Delta SINAD = SINAD' - SINAD = -10 \cdot \text{Log}_{10}\left(10^{-\frac{SINAD}{10}} + 10^{-\frac{SNR}{10}}\left(10^{-\frac{\Delta P}{10}} - 1\right)\right) - SINAD$$

[7] – SINAD adjustment for noise degradation only

[8] makes sense because the premise is that noise does not grow when the signal is made larger and therefore, as shown in [5], the signal-to-noise ratio is directly improved by the amount ΔP and since $SINAD = SNR$, the signal-to-noise-and-distortion ratio improves in the same manner.

Next let's assume that all of the degradation is due to distortion which can only be the case if there is some distortion and there is no noise. This means that the power of the noise is $-\infty$, which means according t [1] that $SNR = \infty$ and according to [3] that $SDR = SINAD$.

Therefore, according to [6], the adjustment for a full-scale sinewave is:

$$\Delta SINAD = -10 \cdot \text{Log}_{10}\left(10^{-\frac{SINAD}{10}} + 10^{-\frac{SINAD}{10}}\left(10^{-\frac{\Delta P}{10}} - 1\right)\right) - SINAD = \Delta P$$

[8] – SINAD adjustment for distortion degradation only

[8] makes sense because the premise is that the distortion grows at the same rate as the signal and therefore making the signal larger makes the distortion similarly larger leading to no improvement to signal-to-distortion ratio and therefore no improvement to signal-to-distortion-and-noise ratio when there is no noise.

$$\Delta SINAD = -10 \cdot \text{Log}_{10}\left(10^{-\frac{SINAD}{10}} + 10^{-\frac{\infty}{10}}\left(10^{-\frac{\Delta P}{10}} - 1\right)\right) - SINAD = 0$$

[9] – SINAD adjustment for distortion degradation only

[8] makes sense because the premise is that noise does not grow when the signal is made larger and therefore, as shown in [5], the signal-to-noise ratio is directly improved by the amount ΔP and since $SINAD = SNR$, the signal-to-noise-and-distortion ratio improves in the same manner.

Next let's assume that all of the degradation is due to distortion which can only be the case if there is some distortion and there is no noise. This means that the power of the noise is $-\infty$, which means according to [1] that $SNR = \infty$ and according to [3] that $SDR = SINAD$.

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[7] is the simple-minded and often incorrect adjustment to SINAD for full-scale, while [8] is the most pessimistic adjustment. In fact:

$$0 \le \Delta SINAD \le \Delta P$$