TECHNICAL BRIEF

SPARQ Dynamic Range

Peter J. Pupalaikis
VP & Principal Technologist

Summary

This paper discusses the dynamic range of the SPARQ signal integrity network analyzer and considers the impact of several key specifications. It further compares dynamic range and key specifications of two competitive time-domain instruments and provides derivations and experimental results that support the calculations.

Dynamic Range

The expression for dynamic range of any TDR based measurement instrument can be expressed according to the following equation which is derived in *Appendix A - Derivation of Dynamic* Range: Appendix A - Derivation of Dynamic Range*:*

$$SNR(f,T)=10\cdot \mathrm{Log}\left(\frac{2\cdot A^2\cdot f_{bw}\cdot Fs_{act}\cdot T}{Ta^2\cdot frac\cdot f^2\cdot Fs_{eq}}\right)-Noise_{dBm}+P(f)+2\cdot(C+F)\cdot CM(f)-6$$

[1] – The Dynamic Range Equation

Where:

- f is frequency (GHz)
- T is the amount of time to average over

And where for the SPARQ instrument:

$A=0.2$ V	is the signal step amplitude
$f_{bw}=102.4$ GHz	is the band limit on noise (not band limited so set to Nyquist rate)
$Fs_{act}=2.5$ MS/s	is the effective actual sample rate (actual sample rate is 10 GS/s)
$Ta=45$ ns	is the equivalent time acquisition length – normal pulser mode (5 MHz)
$frac=10$ %	is the fraction of the acquisition containing reflections – nominal
$Fs_{eq}=204.8$ GS/s	is the equivalent time sample rate
$Noise_{dBm}=-46$ dBm	is the trace noise – with no bandwidth limiting (is -50 dBm band limited to 40 GHz)
$P(f)=\frac{f}{40}\cdot 12$ dB	is the pulser response as function of frequency (dB) – approximately a linear rise to +12 dB @ 40 GHz (falls to 0 dB at approximately 65 GHz).
$F=3.5$ dB	is the switch matrix loss (dB @ 40 GHz)
$C=1.6$	is the user cable loss (dB @ 40 GHz)
$CM(f)=-SE\cdot\sqrt{f}-DE\cdot f$	is a cable model that has 1 dB loss at 40 GHz derived in *Appendix B - Calculation of the Cable Model* where:
$SE=0.133$	is the skin-effect loss (dB/sqrt(GHz))
$DE=0.00404$	is the dielectric loss (dB/GHz)

The SPARQ takes one acquisition consisting of 250 hardware averages per second and uses 1 such acquisition (1 second) in preview mode, 10 such acquisitions (10 seconds) in normal mode, and 100 such acquisitions (100 seconds) in extra mode. If we collect all of the constants in this equation, we obtain the following equation for SPARQ dynamic range:

$$SNR_{SPARQ}(f,T) = 67 + 0.272 \cdot f - 0.931 \cdot \sqrt{f} + 10 \cdot \mathrm{Log}(T) - 20 \cdot \mathrm{Log}(f)$$

[2]

Note that in [2], the cable loss is neglected as external cables and cable specifications are generally user determined and therefore dynamic range is typically specified at the instrument ports.

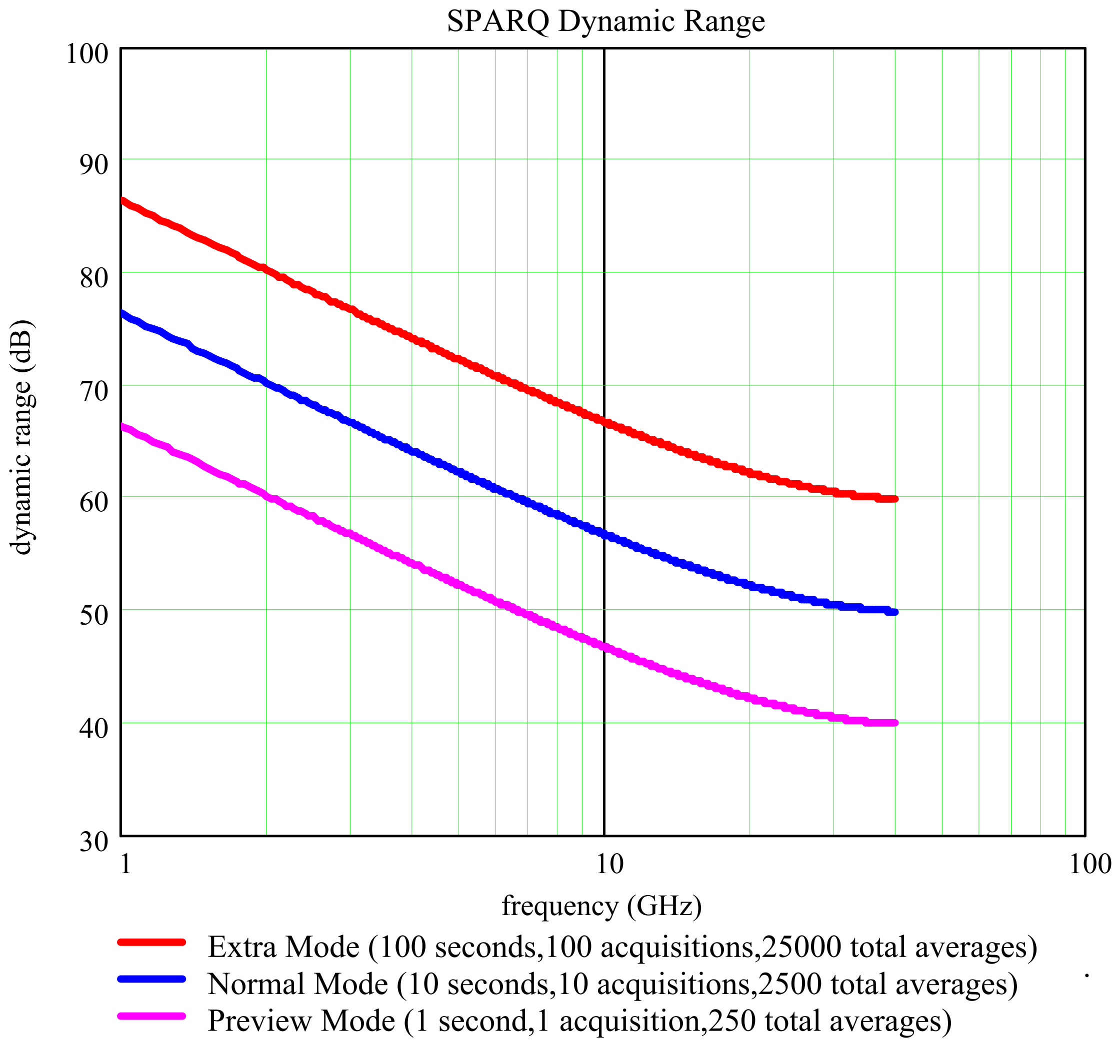

Figure 1 - SPARQ Dynamic Range

Dynamic Range Considerations

The Dynamic range equation [1] has several implications worth discussing. First the obvious ones. Regarding frequency, the dynamic range drops at 20 dB per decade (or 6 dB per octave). This can be considered as the effect of the drop-off in frequency components of a step. If the waveform utilized could be an impulse, this effect could be avoided. This effect is counteracted by the expression $P(f)$ which accounts for practical step responses.

Next is the obvious fact that the dynamic range is strongly dependent on the step size. It goes up by 6 dB for every doubling of the step amplitude, although the high frequency content is also accounted for in $P(f)$ (which is not concerned with the difference between pulser or sampler response).

Dynamic range is directly proportional to the random noise and also losses in the cabling and fixturing, but this is also counteracted by high-sample rate. Dynamic range goes up by 3 dB for every doubling (or 10 dB for every ten times increase) in either the actual sampler sample rate or the time one waits for acquisitions to transpire.

Dynamic range is strongly affected by the length of the acquisition in time as indicated by the squared term Ta in the denominator. The reason why it is squared is two-fold. One effect is the amount of noise let into the acquisition. Remember that the actual signal – the incident wavefront – is contained in a very small time location, yet the noise is spread over the entire acquisition. As the acquisition length increases, the amount of noise increases with no increase in signal. The term $frac$ which accounts for denoising algorithms that serve mainly to restrict the acquisition to areas actually containing signal counteracts this effect. The second effect is the effect on averaging. Longer acquisitions take more time to acquire.

Now some more complicated considerations that are not necessarily obvious. First is the effect of the bandwidth limit on the noise f_{bw}. In many cases, noise in equivalent time sampler arrangements is essentially white. This is especially true if a major source of the noise comes from quantization effects in the ADC. This means that all of the noise power is present up to the Nyquist rate $Fs/2$. In this case, $f_{bw} = Fs_{eq}/2$ and these terms cancel. In this case, the dynamic range is completely independent of the equivalent time sample rate. This may seem counterintuitive because increasing the sample rate causes more noise to fall outside the spectrum of interest due to even noise spreading, but this effect is fully counteracted by the increase in acquisition time and therefore the decrease in the number of acquisitions that can be averaged. In the case where the trace noise is specified with a bandwidth limit (as in most cases), then the dynamic range is actually penalized by $10 \cdot \mathrm{Log}\left(f_{bw}/\left(Fs_{eq}/2\right)\right)$ which seems unfair until you consider that unless the Nyquist rate is set exactly equal to this limit frequency, then acquisitions are needlessly oversampled (needless in theory, not necessarily in practice due to aliasing considerations). To make a proper comparison of band limited and non band limited noise, one must compare using this adjustment.

SPARQ Dynamic Range Technology

SPARQ is designed with several key trade-offs which affect dynamic range. Namely, it is built to be simultaneously low cost and much easier to use. The low cost and ease of use is accomplished partly by utilizing only a single pulser and two samplers, the primary source of cost in TDR based instruments. The ability to utilize a minimum set of pulser/samplers dovetails well with its design for ease of use. This is because the single pulser and each sampler must be able to be connected to each port of the SPARQ during measurements, necessitating a high-frequency switch arrangement. This switch arrangement also provides for the capability to connect to internal standards allowing the unit to calibrate to an internal reference plane without multiple connections and disconnections of the device under test. This internal calibration capability means that the unit is affordable and is much simpler to operate. This capability comes with a large dynamic range price tag because the switch system adds loss to the system and even more importantly, it adds length.

As we see in the dynamic range equation, the losses in the path between the internal pulser/samplers and the ports on the front of the unit contribute twice to the dynamic range reduction because the signal must get from the pulser to the port, go through the DUT and then return from the port to the sampler. In fact, this loss degrades the SPARQ's dynamic range by about 7 dB. But the length is an even bigger penalty because generally the acquisition length must be extended by at least four times the added length. This accounts for a roughly 13 dB reduction in dynamic range. The total price paid for economy and ease of use is approximately 20 dB in dynamic range.

If the design ended there, perhaps SPARQ would simply be a cheaper, less precise way to make measurements, but in fact many special technology features are embodied in the SPARQ to not only counteract the 20 dB reduction, but to provide higher dynamic range than any other TDR based solution.

The first technology is the pulser/sampler response. Most TDR systems utilize a reasonably large amplitude pulse of 250 mV which is flat with frequency. SPARQ uses a similarly sized pulse, but provides a pulse response that rises by 12 dB at 40 GHz. In fact, the response of the pulser/sampler passes back through zero dB at around 65 GHz. The pulser and sampler are very high frequency. Others don't provide such pulse responses because the pulse does not look very good – it exhibits about 80-100% overshoot – making it less attractive for older traditional TDR applications where the response to the pulse was examined visually. The SPARQ's primary mission is to provide S-parameters and calibrated time-domain responses and here the need is for dynamic range and precision over visual attractiveness of the raw pulse. This provides a net 12 dB improvement in dynamic range through non-flatness.

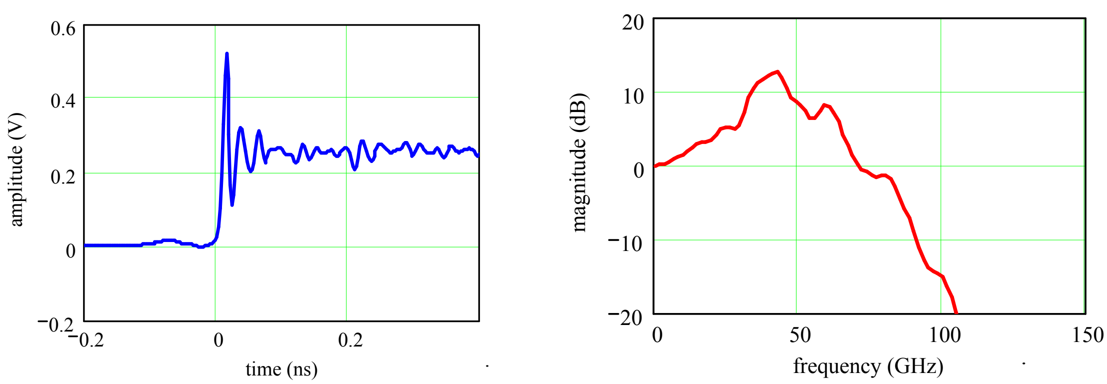

Figure 2 – SPARQ ulta-fast pulser time and frequency domain response

The second technology is the LeCroy patented coherent-interleaved-sampling (CIS) time base. Traditional TDR systems are based on sequential sampling which is very slow and suffers from time base non-linearities. Some even stitch together sequential acquisitions which adds to even more error. The time base nonlinearity will not be seen as a dynamic range degradation but rather as an accuracy degradation. The LeCroy CIS timebase produces a sample clock that is slightly offset from 10 MS/s as it samples the repeating 5 MHz repeating TDR pulse. This allows for much higher sample rates with no time base nonlinearity issues. The CIS time base is not only more precise, it is much simpler to build and operate which enables fast averaging to be performed in hardware. The speed of the sampling system accounts for a 12-18 dB improvement over other sequentially sampled instruments. Unfortunately, when samplers are utilized at higher sample rates, sampler noise increases also, so the net improvement is perhaps only 6 dB or so when factoring in the increased sampler noise. As a final note, because this advantage relies heavily on fast averaging, it is important in the design of the instrument to ensure that this averaging truly results in increased dynamic range. Measurements of this are shown in *Appendix C - Effects of Averaging*

The final key technology is a method of removing noise using digital signal processing techniques called *wavelet denoising*. Techniques like this are used in radar, imaging, and electrocardiogram systems. The effects of this technique are hard to quantify exactly, but the simplest way to look at it is that it counteracts the effects of making the acquisition duration longer to the extent that the algorithm removes noise wherever reflections (i.e. signal) is not present. For short devices this results in a 10 dB improvement or so in dynamic range – for longer devices the improvement can be extreme. When ports of a device exhibit extremely large isolation, wavelet denoising techniques result in dynamic range improvements far in excess of 10 dB.

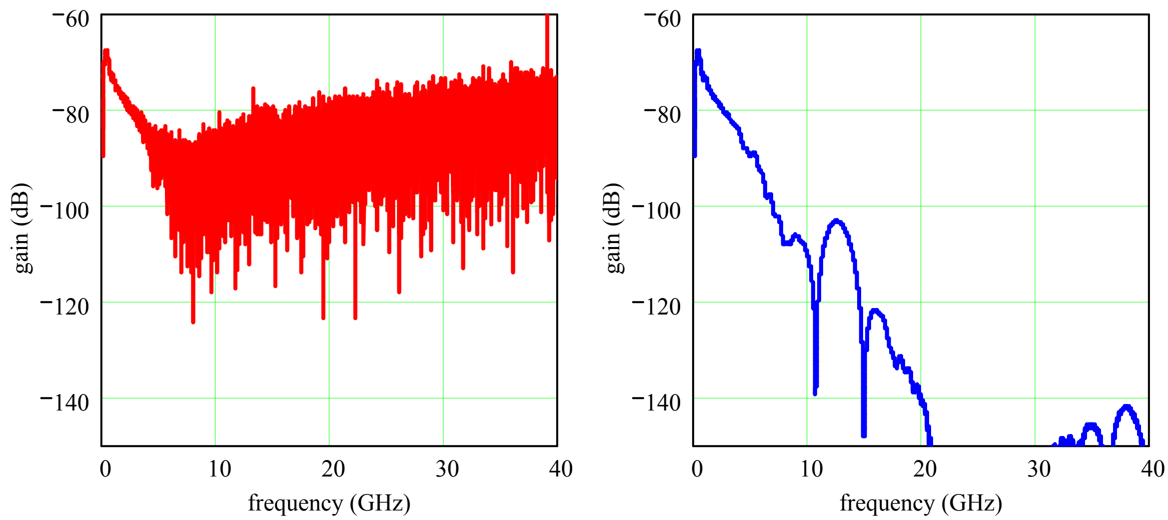

Figure 3 – Comparison of non-denoised (left) and wavelet denoised crosstalk measurement

These three technologies allow a SPARQ to gain conservatively 26 dB improvements in dynamic range allowing it to trade back 20 dB for its lower cost and ease of use and still maintain a 6 dB improvement. This is half the noise or double the frequency, whichever way you want to look at it.

The comparison of SPARQ dynamic range is quantified and compared to the specifications of competitive instruments in the following section. Note that the final dynamic range calculated can be read directly from the competitor data sheets and that the calculations shown here are either exact or even more optimistic than specified for competitive instruments.

Comparison with Other TDR Based Solutions

Specifications	Calculation of Dynamic Range Score	LeCroy SPARQ	Tektronix[1] DSA8200	Agilent 86100C[2]
Normalized Starting Point[3,4,5]	$10\cdot \text{Log}\left(\frac{2\cdot 0.25^2\cdot 40\cdot 150\cdot 10}{45^2\cdot 40^2\cdot 200}\right)-\ldots$ $-50-6+30=24.6$	25 dB	25 dB	25 dB
Sample Rate	$+10\cdot \text{Log}\left(Fs_{act}\right)-52$	10 MS/s Coherent Interleaved Sampling (CIS) (2.5 MS/s effective)[6] 2500 averages **+12 dB**	150 KS/s Sequential Sampling 150 averages **+0 dB**	40 KS/s Sequential Sampling 40 averages **-6 dB**
Noise (@ 40 GHz Bandwidth)	$+Noise_{dBm}-50$	700 uV (-50 dBm)[7] **+0 dB**	370 uV (-56 dBm)[8] **+6 dB**	630 uV (-51 dBm)[9] **+1 dB**
Response @ 40 GHz	$+P(40)$	+ 12 dB[10] **+12 dB**	0 dB **+0 dB**	0 dB **+0 dB**
Step Amplitude	$12+20\cdot \text{Log}(A)$	200 mV **-2 dB**	250 mV **+0 dB**	200 mV[11] **-2 dB**
Losses module to port @ 40 GHz	$-2\cdot F$	3.5 dB[12] **-7 dB**	0 dB **+0 dB**	0 dB **+0 dB**
Electrical Length module to port[13]	$+10\cdot \text{Log}\left(\frac{50}{50-L\cdot 4}\cdot L\cdot 4\right)$	3.6 ns **+0 dB**	0 ns **+13 dB**	0 ns **+13 dB**
Denoising	$+10\cdot \text{Log}(frac)$	Wavelet Hard Threshold[14] **+10 dB**	None **+0 dB**	None **+0 dB**
Total		**50 dB**	**44 dB**[15]	**31 dB**[16]

Table 1 - Comparison of TDR Dynamic Range

1. Utilizes Tektronix DSA8200 sampling oscilloscope and 80E10 TDR module as specified by Tektronix for 40 GHz measurements.
2. Utilizes Agilent 86100C sampling oscilloscope and 54754A TDR module and PSPL 4020 NLTS and 86118A sampler module as specified by Agilent for 40 GHz measurements.
3. Assumes step amplitude of 250 mV, 150 KS/s actual sample rate, -50 dBm noise (bandlimited to 40 GHz), 0 dB response at 40 GHz, no losses from module to port, 50 ns acquisition length, frequency of 40 GHz, 10 seconds of acquisitions averaged, 200 GS/s equivalent time sample rate. Result is rounded up to 25.
4. Note that 80 GS/s sample rate is utilized since all noise specifications are provided bandlimited to 40 GHz. The dynamic range equation shows SNR improving with sample rate which is not the case when noise is band limited. The appropriate setting of sample rate for band limited noise is twice the band limit.
5. Note that 45 ns is used for acquisition length, even though 50 ns has been specified. Half a division is utilized for edge placement.
6. Although CIS is much faster, only 25% of samples acquired are used due to the fact that CIS acquires a full cycle of the 200 ns pulse repetition period. Other advantages to CIS are timebase linearity, but this benefit not quantified in dynamic range.
7. Spec is -46 dBm white noise to Nyquist rate of 100 GHz which is 4 dB improvement when bandlimited to 40 GHz – validated by measurement.
8. Uses typical specification of 370 uV which is much smaller than guaranteed specification of 480 uV
9. Spec is 700 uV at 50 GHz bandwidth so assume 1 dB improvement when bandlimited to 40 GHz
10. Gain is supplied at expense of pulse overshoot specs which are irrelevant in SPARQ – actual pulse frequency response 0 dB at typically 65 GHz
11. Datasheet indicates 4020 produce 200 mV output step amplitude equal to input strobe amplitude.
12. Losses due to cables and switches that facilitate internal calibration capability.
13. The SPARQ has about 3.6 ns electrical length which requires 14.4 extra ns of acquisition. This correction credits competitive units in both averaging time and noise from this extra acquisition length.
14. Patent Pending, assumes 10% of waveform actually contains reflections, which is very conservative.
15. This number can be read almost directly from Tektronix specification. Tektronix shows 45 dB at 250 averages at 40 GHz. The averaging used in this compare is 10 seconds which, with the length credit, is 210 averages which accounts for the 1 dB.
16. Agilent shows 20 dB at 32 GHz with 64 averages. We compare 40 averages, but considering the electrical length module to port, we compare 56 averages. Degrade further by an extra 2 dB to account for the fact that our numbers are at 40 GHz, extrapolates to 18 dB according to Agilent supplied dynamic range data. We cannot explain the deviation between the 31 dB calculated here and Agilent's supplied data of 18 dB.

Appendix A - Derivation of Dynamic Range

Within the SPARQ, we acquire step waveforms, therefore we start with an acquired signal defined as follows:

$$w_k = s_k + \varepsilon_k$$

[3]

where w is the step waveform actually acquired, s is the step portion contain the signal of interest, and ε is the noise signal which we assume to be white, normally distributed, uncorrelated noise.

The signal content in the step is in the form of the frequency content of the derivative, so the derivation must consider this. Since during calculation we don't know the difference between the noise and the step, we must take the derivative of both. We will be approximating:

$$\frac{d}{dt}w(t) = \frac{d}{dt}\big(s(t) + \varepsilon(t)\big) = \ldots$$
$$\frac{d}{dt}s(t) + \frac{d}{dt}\varepsilon(t) = x(t) + \frac{d}{dt}\varepsilon(t)$$

[4]

When we convert the two signals we are interested in to the frequency domain:

$$X = DFT(x(t))$$
$$DN = DFT\left(\frac{d}{dt}\varepsilon(t)\right)$$

[5]

We will calculate the dynamic range as a signal-to-noise ratio (SNR) and define this for each frequency as:

$$SNR_n = \frac{X_n}{DN_n}$$

[6]

In order to calculate the SNR, we calculate the frequency content of each of these components separately and take the ratio. We start with the noise component.

Given a noise signal ε which contains only uncorrelated, normally distributed, white noise, it has a mean of 0 and a standard deviation of σ, which is the same as saying it has an root-mean-square (rms) value of σ. We have K points of this signal ε_k, $k \in 0 \ldots K-1$.

If we calculate the discrete-Fourier-transform (DFT) of this noise signal, we obtain $N+1$ frequency points $N = \frac{K}{2}$, $n \in 0 \ldots N$:

$$E_n = \frac{1}{K}\sum_k \varepsilon_k e^{-j2\pi\frac{nk}{K}}$$

[7]

where the frequencies are defined as:

$$f_n = \frac{n}{N}\frac{Fs}{2}$$

[8]

Where Fs is the sample rate.

By the definition of the rms value and by the equivalence of noise power in the time domain and frequency domain, we know the following:

$$\sqrt{\frac{1}{K}\sum_k {\varepsilon_k}^2} = \sigma = \sqrt{\sum_{n=0}^{N_{bw}}\left(E_n \cdot \frac{2}{\sqrt{2}}\right)^2}$$

[9]

N_{bw} is the last frequency bin containing noise due to any band limiting of noise effects.

We define an average value of Ea that satisfies this relationship:

$$\sqrt{\sum_n\left(Ea \cdot \frac{2}{\sqrt{2}}\right)^2} = \sigma = \sqrt{N_{bw} \cdot \left(\frac{Ea \cdot 2}{\sqrt{2}}\right)^2}$$

[10]

And therefore:

$$Ea = \frac{1}{\sqrt{\frac{f_{bw}}{Fs/2}}} \cdot \frac{\sigma}{\sqrt{K}}$$

[11]

f_{bw} is the frequency limit for the noise calculating by substituting N_{bw} for n in [8].

We, however, are taking the derivative of the signal. The derivative in discrete terms is defined as:

$$\frac{d}{dt}\varepsilon(t) \approx d\varepsilon_k = \frac{\varepsilon_k - \varepsilon_{k-1}}{Ts}$$

[12]

Where $Ts = \frac{1}{Fs}$ is the sample period. Using the same equivalence in [9] and defining $DN = DFT(d\varepsilon)$, we have:

$$\sqrt{\frac{1}{K}\sum_k {d\varepsilon_k}^2} = \sigma = \sqrt{\sum_n\left(DN_n \cdot \frac{2}{\sqrt{2}}\right)^2}$$

[13]

Using the Z-transform equivalent of the derivative in the frequency domain, and an average value for the noise in DN it can be shown that:

$$\sqrt{\frac{1}{K}\sum_k {d\varepsilon_k}^2} = \sigma = \sqrt{\sum_n\left(\frac{\left|1 - e^{-j2\pi\frac{f_n}{Fs}}\right|}{Ts} \cdot Ea \cdot \frac{2}{\sqrt{2}}\right)^2}$$

[14]

Therefore, the average noise component at each frequency is given by:

$$\overline{DN_n} = \frac{\left|1 - e^{-j2\pi \frac{f_n}{Fs}}\right|}{Ts} \cdot Ea \cdot \frac{2}{\sqrt{2}}$$

[15]

We can make an approximation that allows one to gain further insight by expanding the numerator term in a series expansion:

$$\left|1 - e^{-j2\pi \frac{f}{Fs}}\right| = \frac{2\pi f}{Fs} + O\left(\left(\frac{f}{Fs}\right)^3\right)$$

[16]

Which allows us to approximate the noise component as:

$$\frac{\frac{2\pi f_n}{Fs}}{Ts} \cdot Ea \cdot \frac{2}{\sqrt{2}} = \overline{DN_n} = \frac{2\pi f_n \sigma \sqrt{2}}{\sqrt{K}\sqrt{\frac{f_{bw}}{Fs/2}}}$$

[17]

Now that we have the noise component of dynamic range, we move to the signal component.

Without regard to the rise time or the frequency response of the step, which we will consider later, we define the signal such that, in the discrete domain, the integral of the signal forms a step:

$$s_k = s_{k-1} + x_k \cdot Ts$$

[18]

x is an impulse such that $x_0 = \frac{A}{Ts} = A \cdot Fs$ and is zero elsewhere such that s forms a step that rises to amplitude A at time zero and stays there. $X = DFT(x)$ and therefore the signal components at each frequency is defined as:

$$X_n = \frac{A}{Ts} = A \cdot Fs$$

[19]

Again, to gain further insight, we define:

$$K \cdot Ts = \frac{K}{Fs} = Ta$$

[20]

Where Ta is the acquisition duration. Therefore:

$$X_n = \frac{A}{Ta}$$

[21]

Using [6], the ratio can therefore be expressed as:

$$SNR_n = \frac{X_n}{DN_n} = \frac{A \cdot \sqrt{K} \cdot \sqrt{f_{bw}}}{Ta \cdot 2 \cdot \pi \cdot f \cdot \sigma \cdot \sqrt{Fs}}$$

[22]

Since these are voltage relationships, we can express the SNR in dB as:

$$SNR_n = 20 \cdot \mathrm{Log}\left(\frac{A \cdot \sqrt{K} \cdot \sqrt{f_{bw}}}{Ta \cdot 2\pi \cdot f \cdot \sigma \cdot \sqrt{Fs}}\right)\ldots$$
$$= 10 \cdot \mathrm{Log}\left(\frac{A^2 \cdot K \cdot f_{bw}}{Ta^2 \cdot 4 \cdot \pi^2 \cdot f^2 \cdot \sigma^2 \cdot Fs}\right)$$

[23]

And using [20], finally:

$$SNR_n = 10 \cdot \mathrm{Log}\left(\frac{A^2 \cdot f_{bw}}{Ta \cdot 4 \cdot \pi^2 \cdot f^2 \cdot \sigma^2}\right)$$

[24]

We would like to express the noise in dBm, so we have:

$$Noise_{dBm} = 20 \cdot \mathrm{Log}(\sigma) + 13.010\ldots$$
$$= 10 \cdot \mathrm{Log}(\sigma^2 \cdot 20)$$

[25]

And therefore:

$$\sigma^2 = \frac{10^{\frac{Noise_{dBm}}{10}}}{20}$$

[26]

Substituting [26] in [24]:

$$SNR_n = 10 \cdot \mathrm{Log}\left(\frac{A^2 \cdot f_{bw} \cdot 20}{Ta \cdot 4 \cdot \pi^2 \cdot f^2 \cdot 10^{\frac{Noise_{dBm}}{10}}}\right)\ldots$$
$$= 10 \cdot \mathrm{Log}\left(\frac{A^2 \cdot f_{bw} \cdot 20}{Ta \cdot 4 \cdot \pi^2 \cdot f^2}\right) - Noise_{dBm}$$

[27]

Then, to clean things up, we extract some constants:

$$10 \cdot \mathrm{Log}\left(\frac{20}{8 \cdot \pi^2}\right) = -6$$

[28]

And therefore:

$$SNR_n = 10 \cdot \mathrm{Log}\left(\frac{2 \cdot A^2 \cdot f_{bw}}{Ta \cdot f^2}\right) - Noise_{dBm} - 6$$

[29]

Now let's consider some other factors. First, that there is a frequency response of the pulse, and a frequency response of the sampler. These responses can be aggregated into a single response. Since, in decibels, it is simply the frequency response of the step calculated by taking the DFT of the derivative of the step (isolating only the sampled incident waveform) and calculating in dB, this value can simply be added to the dynamic range:

$$SNR_n = 10 \cdot \text{Log}\left(\frac{2 \cdot A^2 \cdot f_{bw}}{Ta \cdot f^2}\right)\ldots$$
$$- Noise_{dBm} + P(f) - 6$$

[30]

Next, we consider the effects of averaging. Averaging the waveform achieves a 3 dB reduction in noise with every doubling of the number of averages. This leads to an improvement of dynamic range by:

$$20 \cdot \text{Log}\left(\sqrt{avg}\right) = 10 \cdot \text{Log}(avg)$$

[31]

Which allows us to insert this directly into the numerator:

$$SNR_n = 10 \cdot \text{Log}\left(\frac{2 \cdot A^2 \cdot f_{bw} \cdot avg}{Ta \cdot f^2}\right)\ldots$$
$$- Noise_{dBm} + P(f) - 6$$

[32]

We really don't want to consider dynamic range in terms of number of averages and instead to prefer to consider the amount of time we are willing to wait. The amount of averages taken in a given amount of time is given by:

$$avg = \frac{Fs_{act}}{Ta \cdot Fs_{eq}} \cdot T$$

[33]

In [33], we now need to distinguish what is meant by sample rate. Fs_{eq} becomes the equivalent time sample rate and replaces what we previously called Fs. Fs_{act} is the actual sample rate of the system and T is the amount of time over which acquisitions are taken. Substituting [33] in [32], we obtain:

$$SNR_n = 10 \cdot \text{Log}\left(\frac{2 \cdot A^2 \cdot f_{bw} \cdot Fs_{act} \cdot T}{Ta^2 \cdot f^2 \cdot Fs_{eq}}\right)\ldots$$
$$- Noise_{dBm} + P(f) - 6$$

[34]

Next, we consider the losses in the cabling and fixturing between the pulser/sampler and the device-under-test (DUT). Since the cabling and fixturing is coaxial and similar, both of these responses follow a response shape that we call a "cable model" which consists of skin-effect loss that is a function of the square-root of the frequency and a dielectric loss that is a function of frequency. We can fit a curve to the fixturing and cabling that is a function of these effects that has been scaled to have a loss of 1 dB at 40 GHz. This is shown *in*

Appendix B - Calculation of the Cable Mode. This curve can then be applied to the fixture and cable responses defined as a single number at 40 GHz:

$$CM(f) = -SE \cdot \sqrt{f} - DE \cdot f$$

[35]

Where SE is the skin-effect loss in $dB/\sqrt{Hz}$ and DE is the dielectric loss in dB/Hz. Where F and C are the loss in the fixturing and cabling respectively at 40 GHz, the equation for dynamic range becomes:

$$SNR(f) = 10 \cdot \mathrm{Log}\left(\frac{2 \cdot A^2 \cdot f_{bw} \cdot Fs_{act} \cdot T}{Ta^2 \cdot f^2 \cdot Fs_{eq}}\right) \ldots$$
$$- Noise_{dBm} + P(f) + 2 \cdot (C + F) \cdot CM(f) - 6$$

[36]

Considering the fact that the signal must pass through the cabling and fixturing twice.

There is one final consideration. That is the effect of denoising algorithms. Denoising algorithms have the effect of removing broadband noise from the acquisition primarily through means of detecting where uncorrelated noise is present in the signal in time. It is difficult to quantify these effects, but a conservative method considers the fact that the primary noise reduction occurs where there are no reflections. In other words, if we look at a denoised waveform, the primary effect is to remove the noise in the locations in the waveform devoid of reflections. The effect on noise, again conservatively speaking, is to retain only the portion of the waveform that contains reflections. Here, we will assume that the noise remains in these portions. Thinking this way, we can define a variable $frac$ that contains the fractional portion of the acquisition that actually contains reflections relative to the portion that does not. This value is DUT dependent and modifies the acquisition duration Ta. Of course, $frac = 1$ is used when denoising is not employed:

$$SNR(f) = 10 \cdot \mathrm{Log}\left(\frac{2 \cdot A^2 \cdot f_{bw} \cdot Fs_{act} \cdot T}{Ta^2 \cdot frac \cdot f^2 \cdot Fs_{eq}}\right) - Noise_{dBm} + P(f) + 2 \cdot (C + F) \cdot CM(f) - 6$$

[37] – Dynamic Range Equation

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Appendix B - Calculation of the Cable Model

Given a magnitude response M_n for frequency points f_n for $N+1$ frequency points $n \in 0 \ldots N$, we first define the magnitude response normalized to have a gain of d dB at the last frequency point f_N. We define an object function to fit to as:

$$\mathbf{V}_n = 20 \cdot \mathrm{Log}\left(|\mathbf{M}_n| \cdot 10^{-\frac{d}{20 \cdot \mathrm{Log}(|\mathbf{M}_n|)}} \right) \quad [38]$$

The cable mode function is defined as:

$$F(f, \mathbf{x}) = -\mathbf{x}_0 \cdot \sqrt{f} - \mathbf{x}_1 \cdot f = -\mathbf{x}^T \begin{bmatrix} \sqrt{f} \\ f \end{bmatrix} \quad [39]$$

The model is determined by fitting values for $\mathbf{x}$ to $\mathbf{V}$:

$$\mathbf{H}_{n,0} = \frac{\partial}{\partial \mathbf{x}_0} F(f_n, \mathbf{x}) = -\sqrt{f_n} \quad [40]$$

$$\mathbf{H}_{n,1} = \frac{\partial}{\partial \mathbf{x}_1} F(f_n, \mathbf{x}) = -f_n \quad [41]$$

And solving for $\mathbf{x}$:

$$\mathbf{x} = \left(\mathbf{H}^T \mathbf{H}\right)^{-1} \mathbf{H}^T \mathbf{V} \quad [42]$$

An example normalized cable and fit for a SPARQ are shown in Figure 4. Here, the constants are:

$$\mathbf{x} = \begin{bmatrix} SE \\ DE \end{bmatrix} = \begin{bmatrix} 0.133 & \frac{dB}{\sqrt{GHz}} \\ 0.00404 & \frac{dB}{GHz} \end{bmatrix} \quad [43]$$

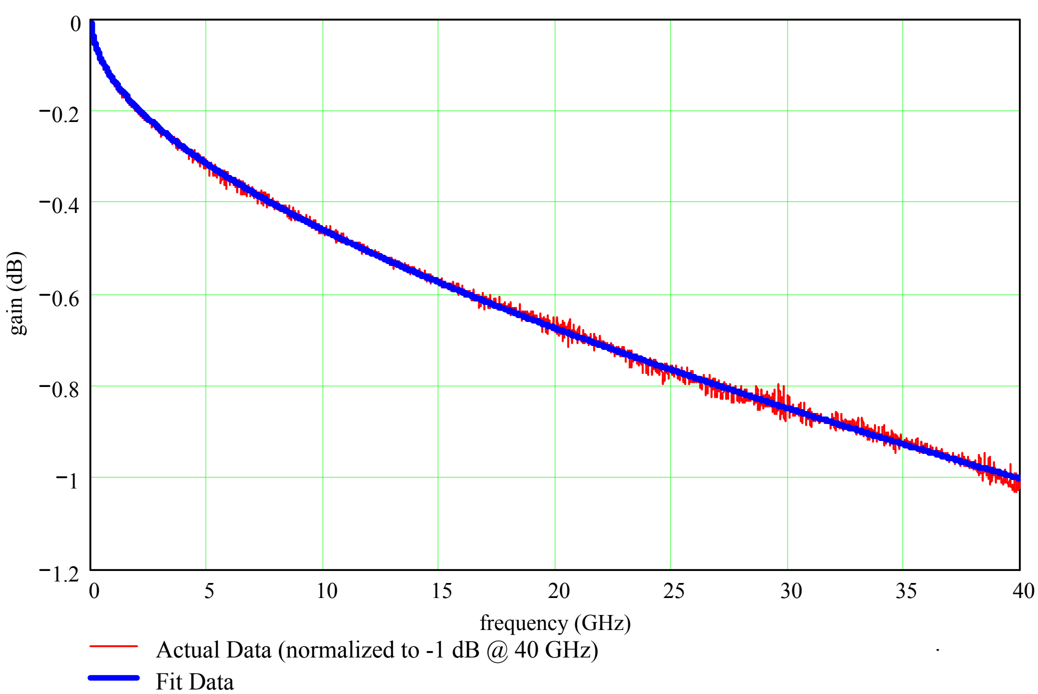

Figure 4 – Fit of Cable Model to Normalized Cable Measurement

Appendix C - Effects of Averaging

In the calculation of dynamic range, there is an assumption that the dynamic range increases by 10 dB if we increase the number of averages by a factor of 10. There are three main operating modes of the SPARQ. These modes are preview, normal, and extra with 250, 2500, and 25000 averages, respectively. Here we show the effects of the averaging in these modes and show that that each mode truly gains 10 dB of dynamic range as we increase the averaging tenfold for each mode improvement. Further, we show the point where averaging produces diminishing returns.

In order to test that averaging truly improves the dynamic range, we set up the SPARQ to compute the standard deviation of a 1 ns portion of the TDR trace while the pulser is turned off and connected to a 50 Ohm termination. Because we need to compute a large data set, we record the cumulative average of 300 acquisitions with no hardware averaging, the cumulative average of 100 acquisitions with 250 hardware averages per acquisition, and the cumulative average of 60 acquisitions with 10,000 hardware averages per acquisition. These sets of data are taken using internal software tools to avoid the fact that most of the tools that are exposed to the user have a limit of 16 bits of precision.

These three sets of data are aggregated and sorted into one large set of data that contains from 1 to 600,000 averages.

We then fit the data to the following function:

$$F(avg, A) = \sqrt{\left(\frac{A_0}{\sqrt{k}}\right)^2 + A_1^{\ 2}} \quad [44]$$

The motivation for this function is as follows:

- A_0 contains the uncorrelated noise (noise that can be averaged away).
- A_1 contains the correlated noise (noise that cannot be averaged away).

Ideally, A_0 is as low as possible to reflect a low noise sampler, A_1 is zero meaning that continued averaging improves the noise forever.

When we fit the data we obtain the following values:

- $A_0 = 1.131\ mV$ or $-45.922\ dBm$
- $A_1 = 2.623\ \mu V$ or $-98.615\ dBm$

The variance is $3.009e-12$.

Note that the noise used for a SPARQ sampler is not shown band limited and improves by 4 dB to -50 dBm when band limited to 40 GHz.

The result of the fit is shown in Figure 5 where it is seen that all of the SPARQ averaging modes have a 10 dB noise improvement and that in fact another factor of 10 beyond extra mode would bring the noise close to the best noise achievable.

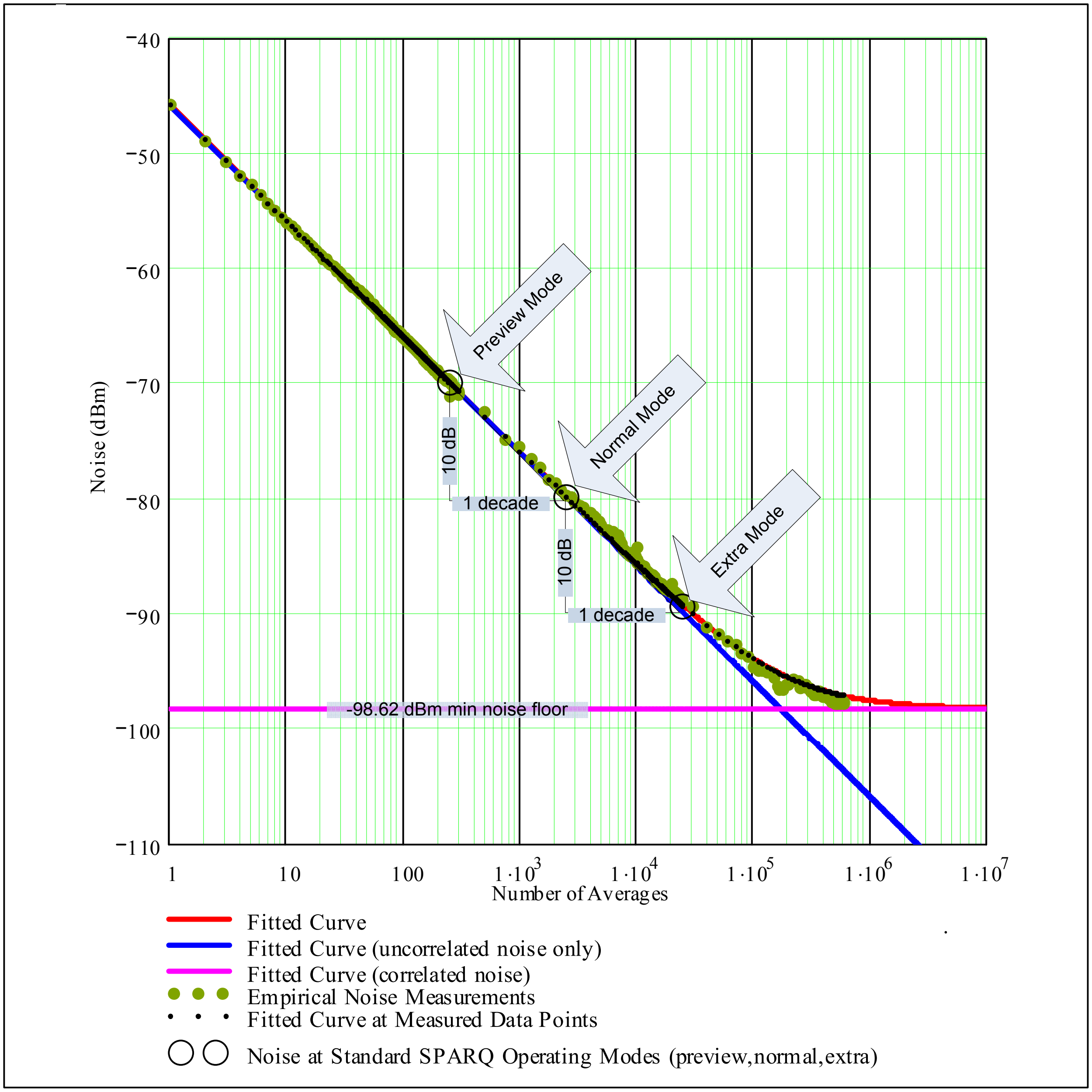

Figure 5 – Effect of Averaging on Noise

Appendix D – Coherent Interleaved Sampling Mode (CIS) Calculations

Fs_{eq} is the equivalent time sample rate (GS/s).

$SP = 2048$ is the samples per pattern specified for the CIS timebase:

$$Fs_{eq} = \frac{SP}{10} = 204.8 \text{ GS/s} \quad [45]$$

This corresponds to a Nyquist rate of $Fs_{eq}/2 = 102.4$ GHz and a sample period of $Ts = 1/Fs_{eq} = 4.883$ ps.

We have two modes of operation, which refer to the electrical length of the DUT. These are normal mode and long mode. Normal mode utilizes a 5 MHz pulser repetition rate and long mode utilizes a 1 MHz pulser repetition rate. The duty cycle is a minimum of 30 percent in both modes, which leads to a maximum acquisition duration of:

$PR = 5$ MHz repetition rate in normal mode.

$avg = 250$ is the number of hardware averages per single CIS mode acquisition.

T is the time spent acquiring (s).

$Fs_{act} = 10$ MS/s is the CIS mode actual sample rate.

The update rate is:

$$T = \frac{avg \cdot Fs_{eq}}{PR \cdot Fs_{act}} = 1 \quad [46]$$

is the acquisition time in seconds for a single, hardware averaged acquisition (250 times).

Since $DC = 30\%$ is the duty cycle of the pulser

$Ta_{\max} = DC/PR = 60$ ns and is the maximum acquisition duration in normal mode.

We use $Ta = 45$ ns as the acquisition duration for normal mode to make sure we fit it – 10 divisions at 5 ns/div minus half a division to position the TDR pulse edge.