

# Practical Considerations in Measuring Power and Efficiency on PWM and Distorted Waveforms during Dynamic Operating Conditions

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# Introduction

- Power Analyzers have long been used to measure power and efficiency in electrical apparatus and power conversion systems
- Most Power Analyzers have been suitably adapted to measure power and efficiency of distorted signals, e.g., pulse-width modulated (PWM) signals.
- However, the measurement and use model of a Power Analyzer is usually such that it is measuring power and efficiency during steady-state (“static”) operating conditions, and technical test standards assume this use model.
- Additionally, there is no known (to me) technical test standard detailing how to measure efficiency during dynamic operating conditions, especially in systems where the two sets of power signals (e.g., input and output) have different cyclic periods.
- This issue is fully described herein, and comments on a dynamic efficiency technique that we have employed are solicited.

# Introduction, cont'd

*What began this discussion within Teledyne LeCroy and with our customer contacts?*

- An efficiency calculation of 112.27% in beta software kicked things off....
- **Question (LeCroy):**  
“We know why this is wrong - what should we do instead?”
- **Answer (Customer):**  
“We don’t know – we’ve never thought of efficiency dynamically before...”



# Background:

## Digital Sampling Power Measurement Technique Using Cyclic Period Detection

The basic techniques for using an analog-to-digital converter (ADC) to sample a waveform and then perform power calculations on the sampled data are described. This technique is required for distorted waveforms, but also works for pure sinusoids. This technique produces substantially the same result on different instrument types with equivalent ADC resolution.



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# “Distorted” Waveforms are Complex Sums of Sinusoids

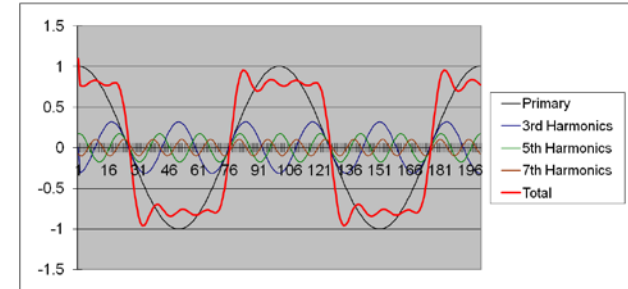
*Therefore, a digital sampling technique is required for measuring power in distorted waveforms*

- Any “distorted” (e.g. PWM) waveform is composed of different amplitudes of odd integer sinusoidal harmonics (“orders”)



Square Wave  
Harmonics

$$x_{\text{square}}(t) = \frac{4}{\pi} \sum_{k=1}^{\infty} \frac{\sin((2k-1)2\pi ft)}{(2k-1)}$$
$$= \frac{4}{\pi} \left( \sin(2\pi ft) + \frac{1}{3} \sin(6\pi ft) + \frac{1}{5} \sin(10\pi ft) + \dots \right).$$

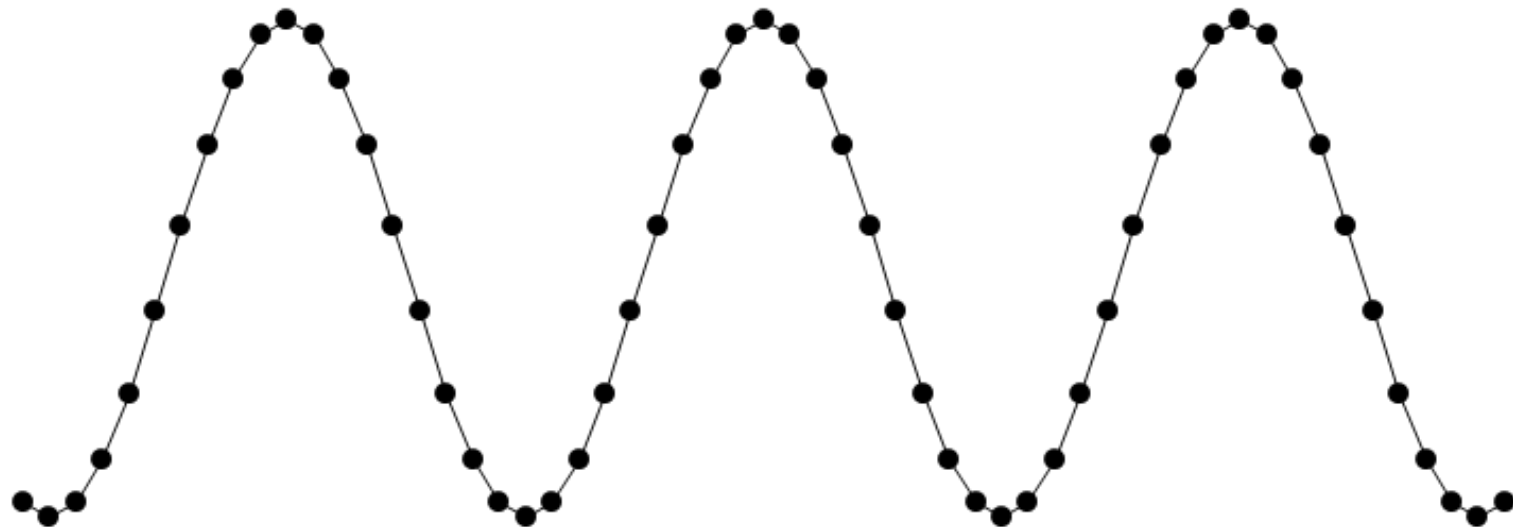


- The voltage and current sinusoid pairs will have different magnitudes for different harmonic orders.
- The phase relationships between voltage and current sinusoid pairs for different harmonic orders is not a constant.
- There is no practical method to measure phase angle between many voltage and current sinusoid pairs, as would be required for distorted signals.
- If phase angle between all voltage and current sinusoid pairs for all harmonics cannot be practically measured, then apparent power, reactive power, and total phase angle (and power factor) cannot be calculated using a “traditional” approach.

# Measurement Step 1 – Digitally Sample the Waveforms

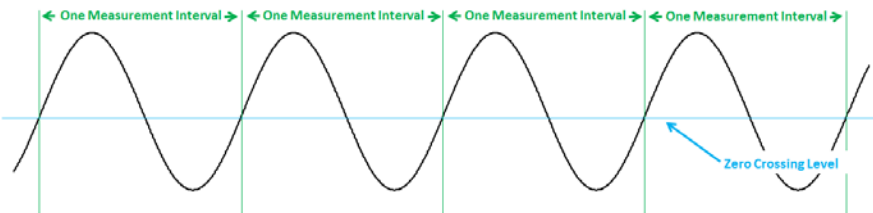
*This approach is commonly used by Power Analyzers and, of course, modern oscilloscopes*

- A “digital” acquisition system samples the analog signal at a given rate (the “sample rate”) that is fast enough to capture all desired signal frequencies (the “Nyquist criterion”)



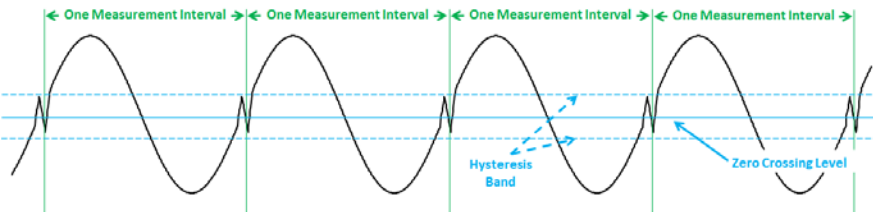
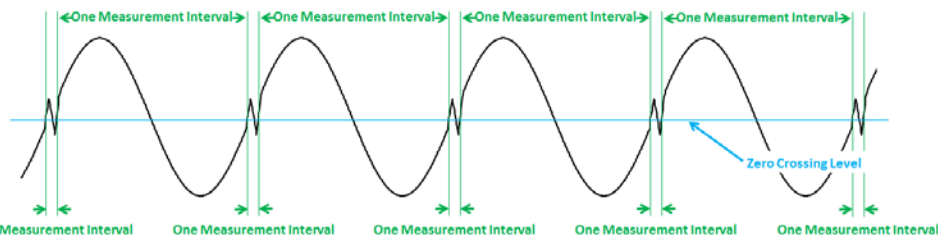
# Measurement Step 2 – Determination of the Cyclic Period

*Hysteresis band settings provide flexibility and improved utility*



A low-pass filter (LPF) is applied and a software algorithm determines the beginning and end of each cyclic period

It is possible that remaining distortion could cause incorrect cyclic period determination...



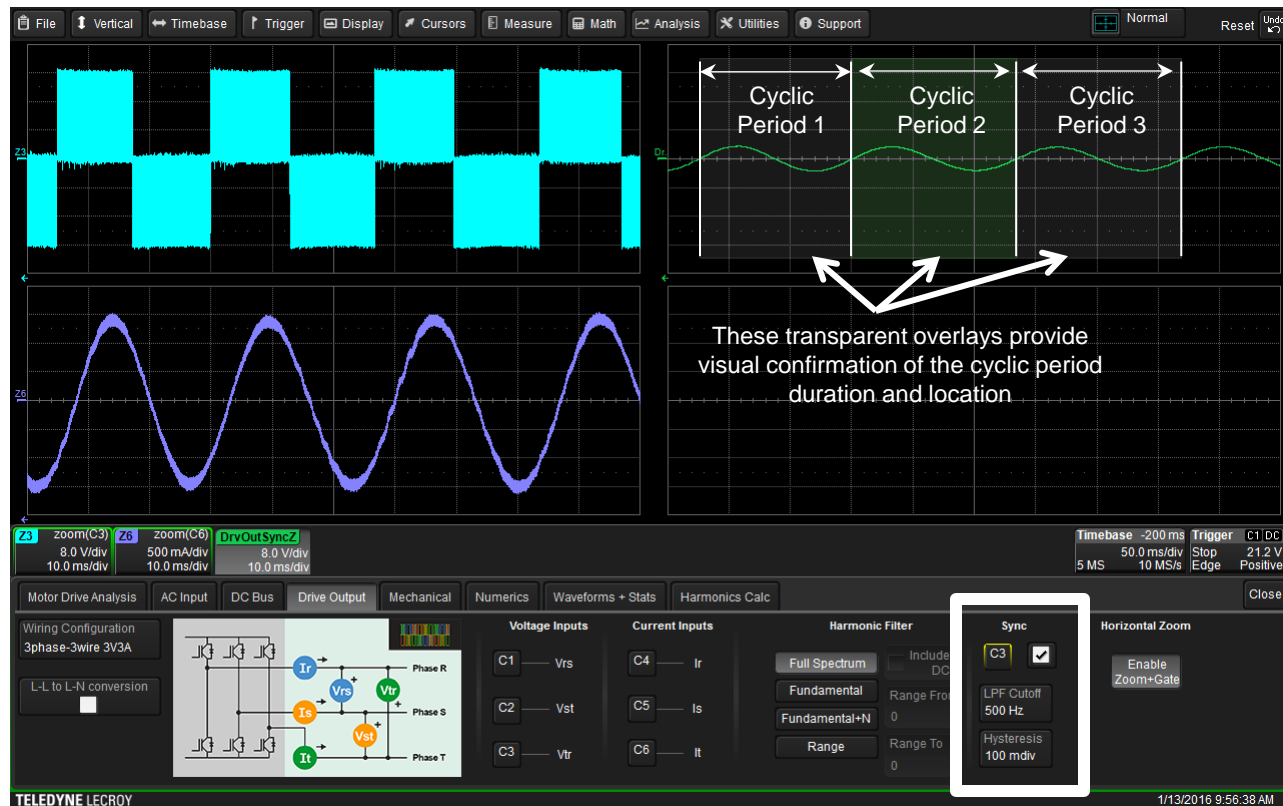
Therefore, hysteresis adjustment is sometimes provided to permit correct calculation of the cyclic periods.

Some simple examples follow to demonstrate that a LPF and Hysteresis Band adjustment can be used for proper cyclic period detection of very distorted signals under highly dynamic conditions. For more examples, see the Teledyne LeCroy Motor Drive Analyzer Software Instruction Manual, <http://cdn.teledynelecroy.com/files/manuals/motor-drive-analyzer-software-operators-manual.pdf>, or <http://teledynelecroy.com/doc/3-phase-power-app-note>

# Determination of Cyclic Period – Example 1

*Sine-modulated PMSM Motor, “Sync” on Line-Line Voltage Signal*

- Default Low-Pass Filter (LPF) Setting
- Default Hysteresis Setting





# Determination of Cyclic Period – Example 2

## *Sine-modulated PMSM Motor, “Sync” on Line Current Signal*

- Default Low-Pass Filter (LPF) Setting
- Default Hysteresis Setting



# Determination of Cyclic Period – Example 3

## Six-Step Commutated BLDC Motor, “Sync” on Line-Line Voltage Signal

- Default Low-Pass Filter (LPF) Setting
- Default Hysteresis Setting



# Determination of Cyclic Period – Example 4

## Six-Step Commutated BLDC Motor, “Sync” on Line Current Signal

- Default Low-Pass Filter (LPF) Setting
- Default Hysteresis Setting



# Determination of Cyclic Period – Example 5

*Dynamic operating condition, sine-modulated PMSM Motor*

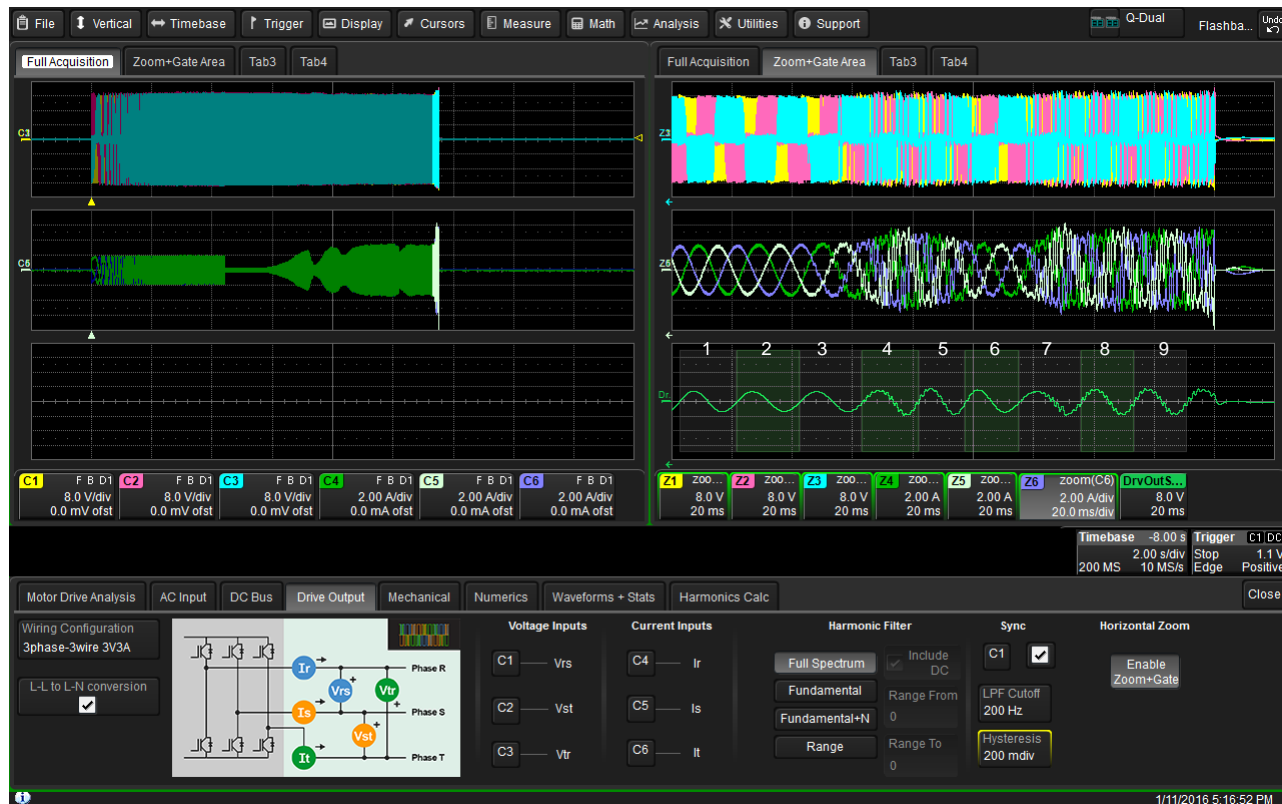
- Default Low-Pass Filter (LPF) Setting
- Default Hysteresis Setting
- Cyclic periods detected incorrectly near overload event



# Determination of Cyclic Period – Example 5

*Dynamic operating condition, sine-modulated PMSM Motor with LPF and Hysteresis changes*

- 200 MHz Low-Pass Filter (LPF) Setting
- 200 mdiv Hysteresis Setting
- **Cyclic periods now detected correctly near overload event**



# Determination of Cyclic Period – Example 5

*Dynamic operating condition, sine-modulated PMSM Motor with LPF and Hysteresis changes*

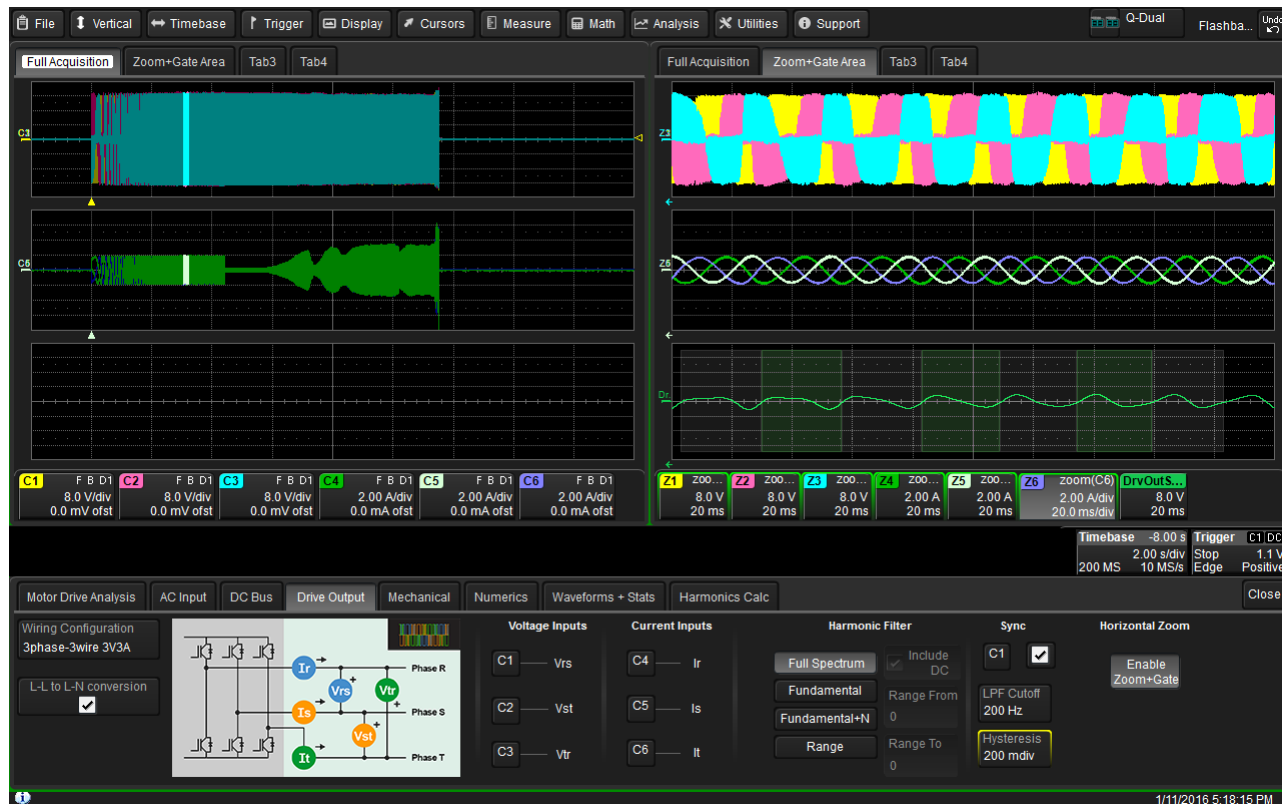
- 200 MHz Low-Pass Filter (LPF) Setting
- 200 mdiv Hysteresis Setting
- Cyclic periods detected correctly during steady-state, no-load condition



# Determination of Cyclic Period – Example 5

*Dynamic operating condition, sine-modulated PMSM Motor with LPF and Hysteresis changes*

- 200 MHz Low-Pass Filter (LPF) Setting
- 200 mdiv Hysteresis Setting
- Cyclic periods detected correctly during start-up condition



# Determination of Cyclic Period – Example 5

*Dynamic operating condition, sine-modulated PMSM Motor with LPF and Hysteresis changes*

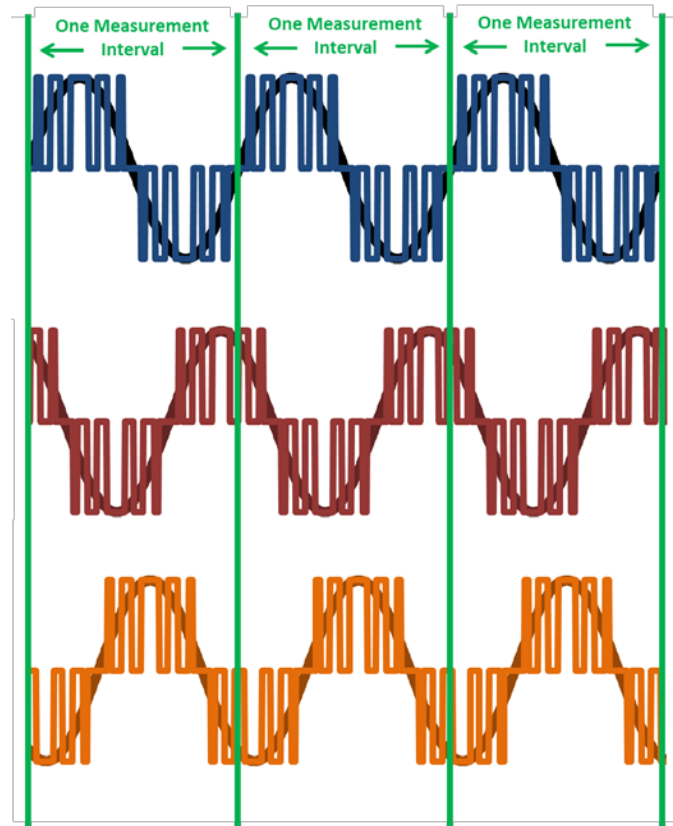
- 200 MHz Low-Pass Filter (LPF) Setting
- 200 mdiv Hysteresis Setting
- Cyclic periods detected correctly during start-up condition





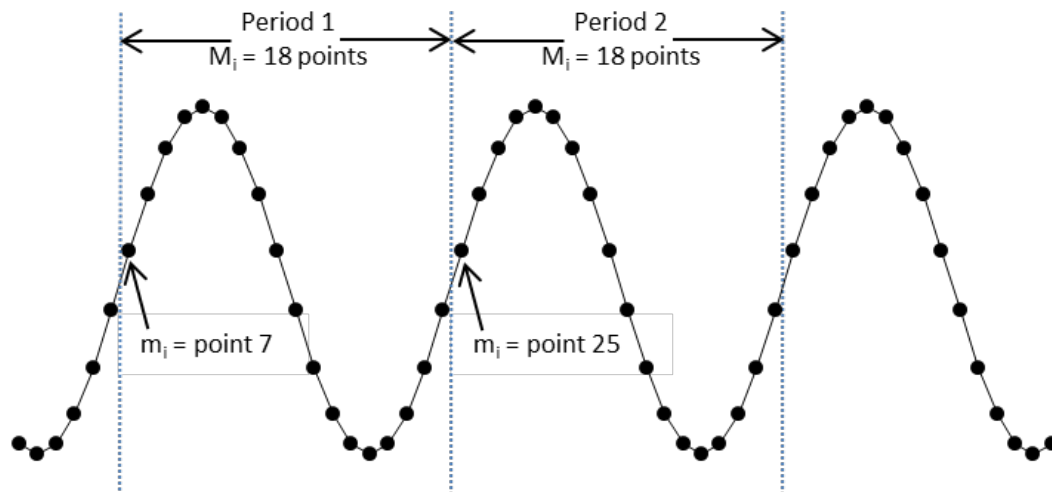
# Measurement Step 3 – Apply Cyclic Period to All Signals

- All acquired voltage, current, or other signals (e.g. mechanical shaft speed, torque, direction, etc.) have the cyclic period time applied to them
- Note: more than one cyclic period may be identified because cyclic periods may substantially differ between waveforms
  - e.g., AC 50/60 line input and variable frequency drive (VFD) output
  - e.g., VFD output and motor mechanical shaft speed/torque sensing.



# Measurement Step 4 – Calculate Per-cycle Values

- The digitally samples in each signal are now grouped into measurement periods (cycles), as determined by the Sync signal.
- For a given cycle index  $i$ ...
- the digitally sample voltage waveform is represented as having a set of sample points  $j$  in cycle index  $i$ ...
- For a given cycle index  $i$ , there are  $M_i$  sample points beginning at  $m_i$  and continuing through  $m_i + M_i - 1$ .
- Voltage, current, power, etc. values are calculated on each cycle index  $i$  from 1 to  $N$  cycles.



## Example

- Period 1 is cycle index  $i = 1$
- There is a set of  $j$  sample points in Period 1, beginning with point 7 and ending with point 24
- All Period 1 voltage, current and power calculations are made with this set of points
- Period 2 is cycle index  $i = 2$
- There is a set of  $j$  sample points in Period 2, beginning with point 25 and ending with point 42
- All Period 2 voltage, current and power calculations are made with this set of points
- And so on through Period  $N$

# Formulas Used for Per-cycle Digitally Sampled Calculations

*“Mean” values are calculated from the per-cycle data set*

	Per-Cycle Calculated Values	Mean Calculated Values
<b>V<sub>RMS</sub></b>	$Vrms_i = \sqrt{\frac{1}{M_i} \sum_{j=m_i}^{m_i+M_i-1} V_j^2}$	$Vrms = \frac{1}{N} \sum_{i=1}^N Vrms_i$
<b>I<sub>RMS</sub></b>	$Irms_i = \sqrt{\frac{1}{M_i} \sum_{j=m_i}^{m_i+M_i-1} I_j^2}$	$Irms = \frac{1}{N} \sum_{i=1}^N Irms_i$
<b>Real Power (P, in Watts)</b>	$P_i = \frac{1}{M_i} \sum_{j=m_i}^{m_i+M_i-1} V_j * I_j$	$P = \frac{1}{N} \sum_{i=1}^N P_i$
<b>Apparent Power (S, in VA)</b>	$S_i = Vrms_i * Irms_i$	$S = \frac{1}{N} \sum_{i=1}^N S_i$
<b>Reactive Power (Q, in VAR)</b>	$\text{magnitude } Q_i = \sqrt{S_i^2 - P_i^2}$ <p><i>sign of Q<sub>i</sub> is positive if the fundamental voltage vector leads the fundamental current vector</i></p>	$Q = \frac{1}{N} \sum_{i=1}^N Q_i$

# Formulas Used for Per-cycle Digitally Sampled Calculations

*“Mean” values are calculated from the per-cycle data set*

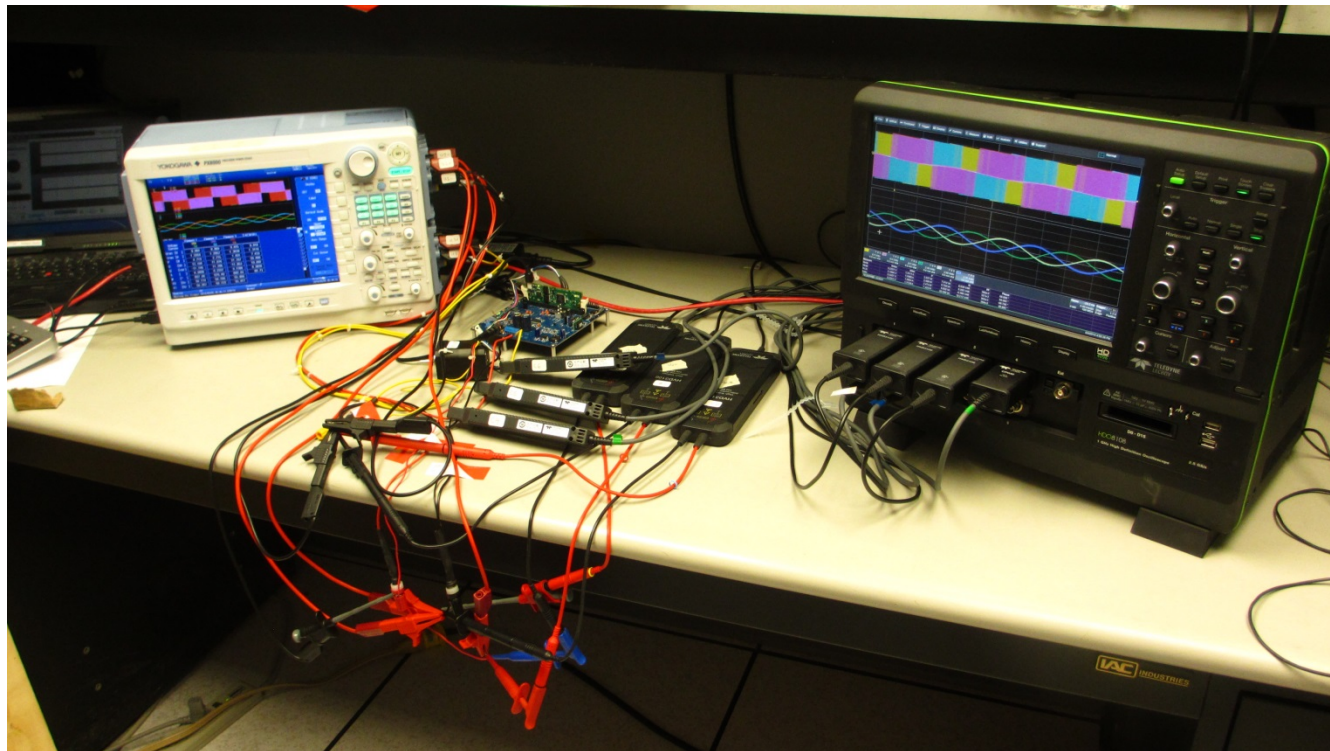
	Per-Cycle Calculated Values	Mean Calculated Values
<b>Power Factor (<math>\lambda</math>)</b>	$\lambda_i = \frac{P_i}{S_i}$	$\lambda = \frac{1}{N} \sum_{i=1}^N \lambda_i$
<b>Phase Angle (<math>\phi</math>)</b>	$\text{magnitude } \phi_i = \cos^{-1} \lambda_i$ <p><i>sign of <math>\phi_i</math> is positive if the fundamental voltage vector leads the fundamental current vector</i></p>	$\phi = \frac{1}{N} \sum_{i=1}^N \phi_i$

These formulas are generalized and can differ somewhat based on the number of phases, the wiring configuration and the number of wattmeters used for three-phase measurements. For complete detail on all calculations, reference the Teledyne LeCroy Motor Drive Analyzer Software Instruction Manual, see <http://cdn.teledynelecroy.com/files/manuals/motor-drive-analyzer-software-operators-manual.pdf>

# Accuracy Comparison

## 12-bit Digital Oscilloscope platform compared to 12-bit Power Analyzer

- Identical Setups for
  - Capture Time
  - Sample Rate
  - Bandwidth Setting
- Same Acquisition (cross-triggered)
- Instruments
  - Teledyne LeCroy MDA810 Motor Drive Analyzer
  - Yokogawa PX8000 Power Analyzer

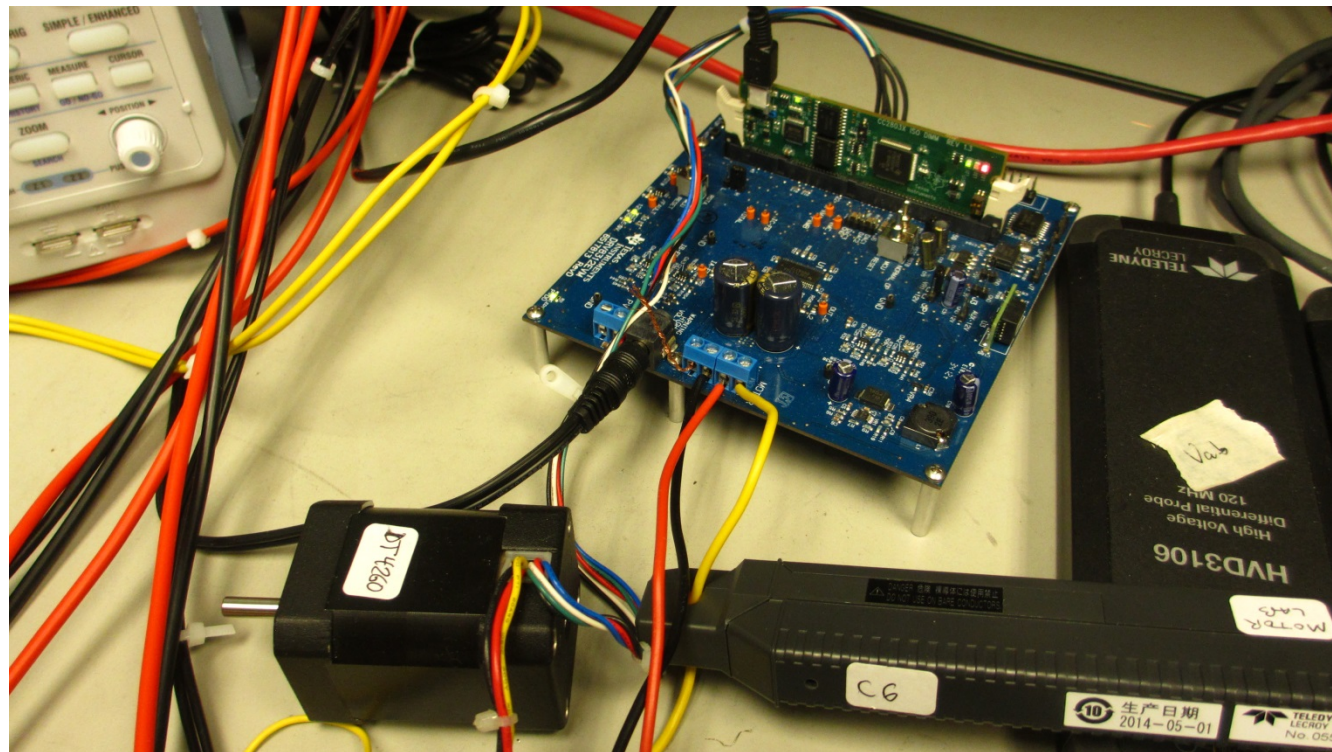




# Accuracy Comparison, cont'd

## 12-bit Digital Oscilloscope platform compared to 12-bit Power Analyzer

- Device Under Test
  - Texas Instruments Motor Drive Evaluation Board
  - PMSM
  - Sine-modulated controls



# Accuracy Comparison, cont'd

## Results Direct Comparison and Comments

	$\Sigma V_{\text{RMS}}$ (L-L)	$\Sigma V_{\text{RMS}}$ (L-N)	$\Sigma I_{\text{RMS}}$	P	S	Q	PF	Phase
Yokogawa	8.549 V	4.924 V	1.271 A	6.438 W	18.777 VA	17.638 VAR	.3429	69.95°
Teledyne LeCroy	8.6063 V	4.9566 V	1.2619 A	6.406 W	18.764 VA	17.636 VAR	.341	70.036°
% Difference	+0.67%	+0.66%	-0.72%	-0.49%	-0.07%	-0.01%	-0.55%	+0.12%

- Notes:
  - The Teledyne LeCroy MDA810 is using voltage and current probes, whereas the Yokogawa PX8000 is using direct voltage and current inputs
- Results are very close between the two instruments
  - Close correlation between the oscilloscope platform and the power analyzer would not have been possible using an 8-bit oscilloscope

# Measurement of Static and Dynamic System Behaviors

Typical approaches and equipment used to measure electrical and mechanical power behaviors during “static” steady-state operation will be reviewed, and “dynamic” measurement definitions, techniques and measurements will be introduced.





# “Static” Power and Efficiency Analysis of Electric Motors

- Dynamometer Test Stand
  - Power analyzer typically used for power measurements
  - Dynamometer applies known load
- Test Validation and Reporting
  - Power measurements made at a single speed, load, torque, temperature, etc. condition
  - Operating curves are derived from compilation of separate static test events (e.g. Torque vs. Speed, Efficiency vs. Speed)
- Generally not an integrated R+D test
  - Conceived to validate power and efficiency performance of larger motors (10% of unit volume) that consumed >90% of electricity
  - Efficiency standards are (mostly) written around this test paradigm



# Static Power Analysis

*Both instrument solutions provide calculated mean power values in a table*



Teledyne LeCroy Motor Drive Analyzer

$$P = \frac{1}{N} \sum_{i=1}^N P_i$$

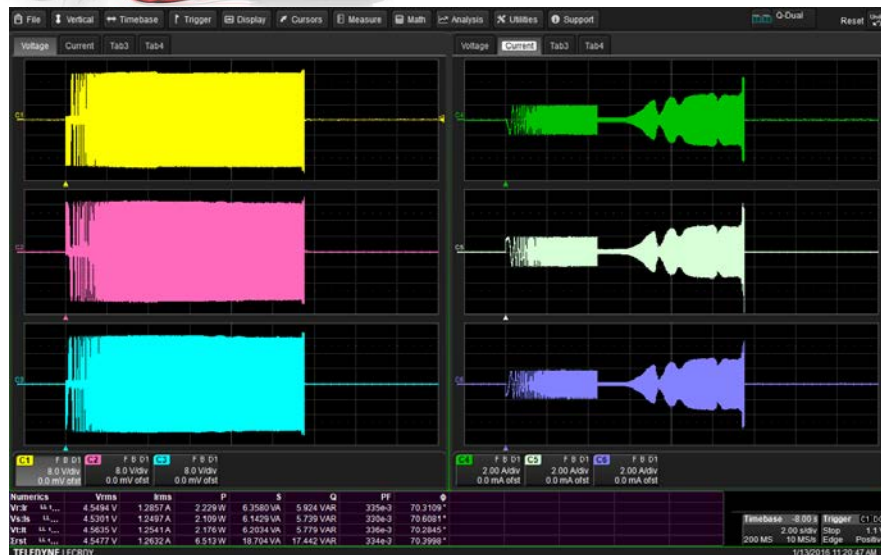
Typical Power Analyzer



# Dynamic Power Analysis

The Teledyne LeCroy solution also provides per-cycle calculations during dynamic events

Teledyne LeCroy Motor Drive Analyzer



## Static Power

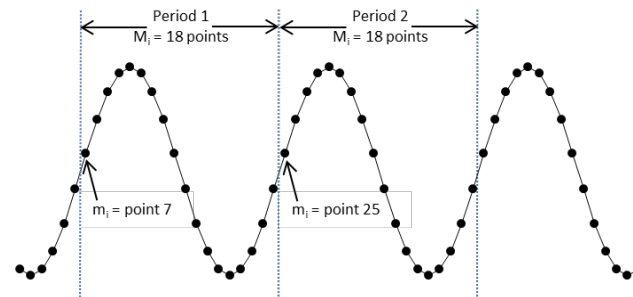
$$P = \frac{1}{N} \sum_{i=1}^N P_i$$

One mean value per acquisition time period.

## Dynamic Power

$$P_i = \frac{1}{M_i} \sum_{j=m_i}^{m_i+M_i-1} V_j * I_j$$

One value per cycle.  
“N” values per mean value for one acquisition time period.



# Static and Dynamic Power Analysis - Summary

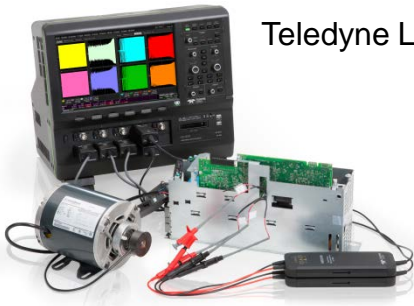
*Nearly identical capabilities for Static Analysis...But what should be done for Dynamic Analysis?*

Capability	Teledyne LeCroy Motor Drive Analyzer	Power Analyzer Instrument
Static Power Analysis	Yes Short records. Constant load/speed Mean calculated values	Yes Short records Constant load/speed Mean calculated values
Dynamic Power Analysis	Yes Long time durations Variable loads/speeds Per-cycle calculations <u>Unmatched cyclic periods</u>	Not in one acquisition record To my knowledge...

- Variable Frequency Drives can have line (50/60 Hz) inputs with variable frequency outputs.
- During steady-state “static” operating conditions, efficiency calculations using mean input and output power values would produce a valid mean efficiency value (one efficiency value per acquisition period)
- **During “dynamic” operating conditions, how should efficiency be calculated dynamically (per-cycle)?**

# Efficiency Analysis Formula and Approach Comparison

*Static efficiency calculates substantially the same in both cases...*



Teledyne LeCroy Motor Drive Analyzer

Typical Power Analyzer



Static Efficiency	Dynamic Efficiency	Static Efficiency	Dynamic Efficiency
$\eta = \frac{1}{N} \sum_{i=1}^N \eta_i$	$\eta_i = \left( \frac{P_{2i}}{P_{1i}} \right) * 100\%$	$\eta = \frac{P_2}{P_1}$	Simple compilation of multiple static measurements made under different operating conditions
One mean value per acquisition time period.	One value per cycle. “N” values per mean value for one acquisition time period.	One mean value per acquisition time period.	—



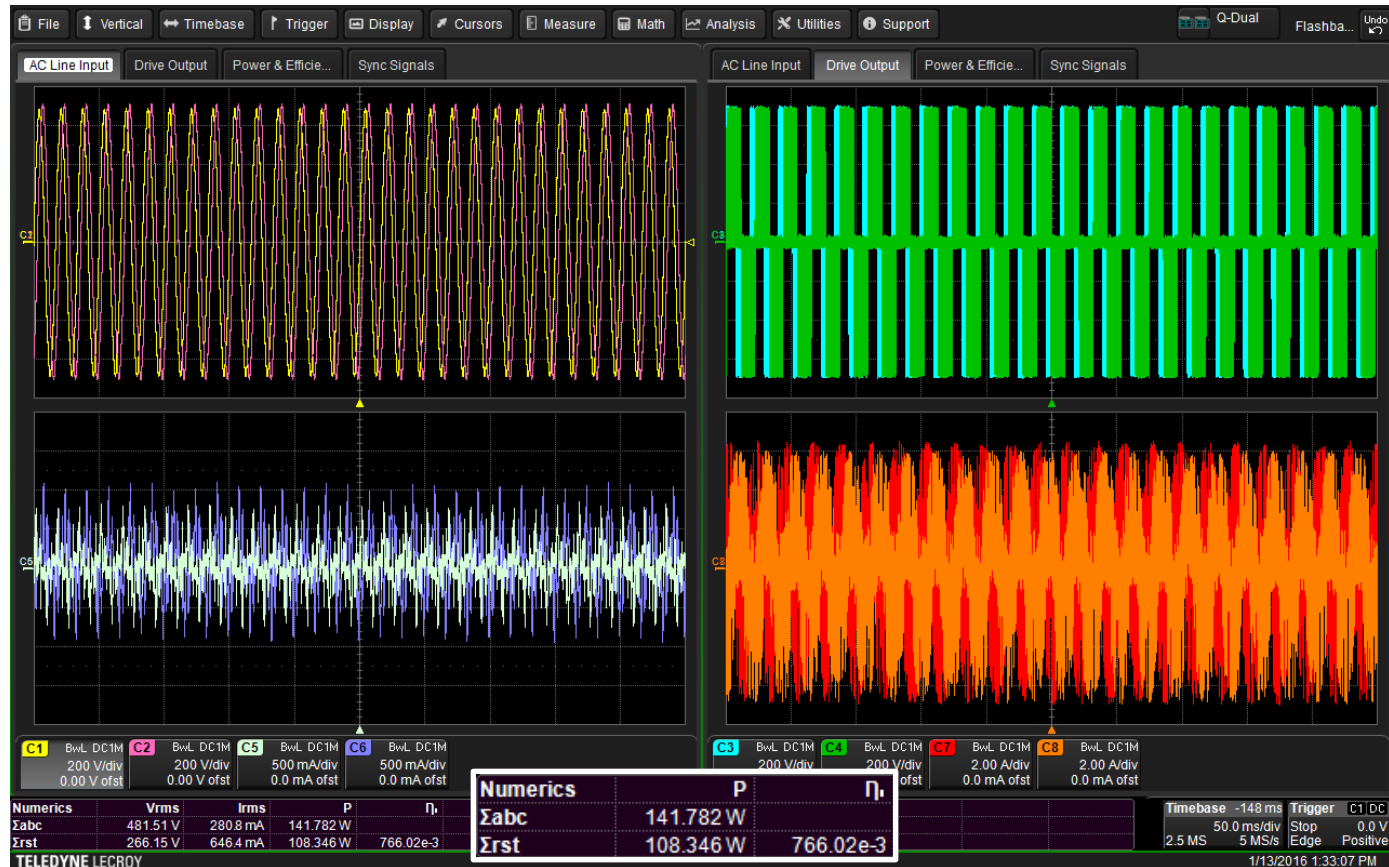
# Example and Detailed Comparison: 480V Variable Frequency Drive Input-Output **Static** Efficiency Analysis

This examples demonstrates the approach taken to measure per-cycle power and efficiency during a “static” (steady-state) condition when the input and output power frequencies are not the same. This example shows the use of the Teledyne LeCroy instrument, but substantially the same numeric mean value efficiency result would be obtained with any suitable instrument.



# Static Power and Efficiency Analysis – AC Line Input to Drive Output

500ms acquisition, 2 wattmeter method. Note that Efficiency  $\neq P(\Sigma RST)/P(\Sigma ABC) \dots$  Why?



# Per-cycle Efficiency Waveform and Sync Waveforms with Overlay

One Sync signal (middle) is the AC Input and the other (bottom) is the Drive Output

Per-cycle  
Efficiency vs.  
Time  
calculated  
waveform

50 Efficiency  
Calculations

29  
AC Input  
Sync Periods

22  
Drive Output  
Sync Periods



Mean Value  
Numerics Table  
Complete  
Statistics for 50  
Efficiency  
calculations

**Note:** Waveform trace thickness has been enhanced in this image to improve viewing on a projector



# Per-cycle Efficiency, Power( $\Sigma ABC$ ) and Power( $\Sigma RST$ ) Waveforms

Sync signals with overlays are in the same grid as their associated power waveforms



50 Efficiency Calculations

29 AC Input Sync Periods

22 Drive Output Sync Periods

Numerics	P	$\eta$
$\Sigma abc$	141.782 W	
$\Sigma rst$	108.346 W	766.02e-3

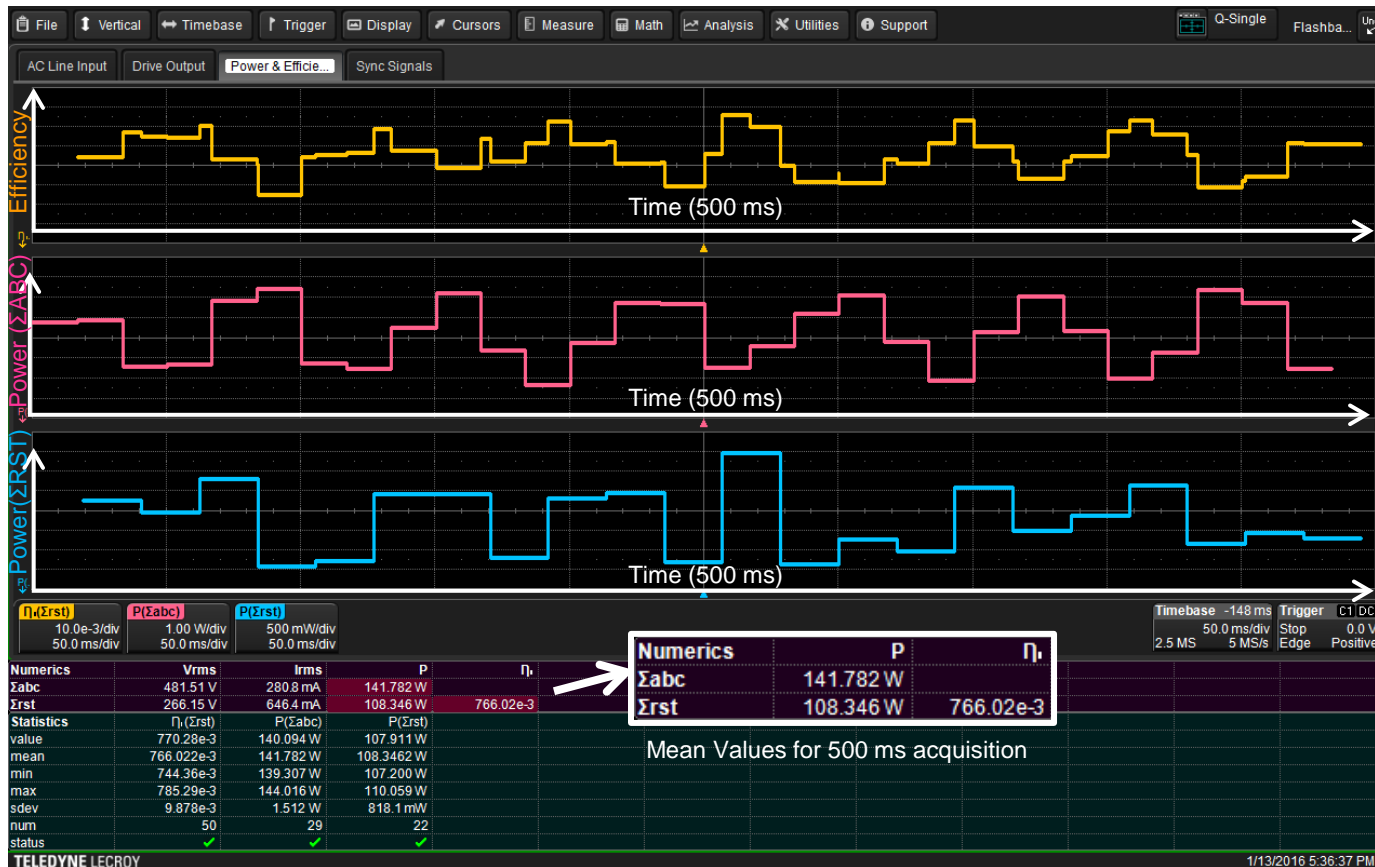
Mean Values for 500 ms acquisition

**Note:** Waveform trace thickness has been enhanced in this image to improve viewing on a projector



# Per-cycle Efficiency, Power( $\Sigma ABC$ ) and Power( $\Sigma RST$ ) Waveforms

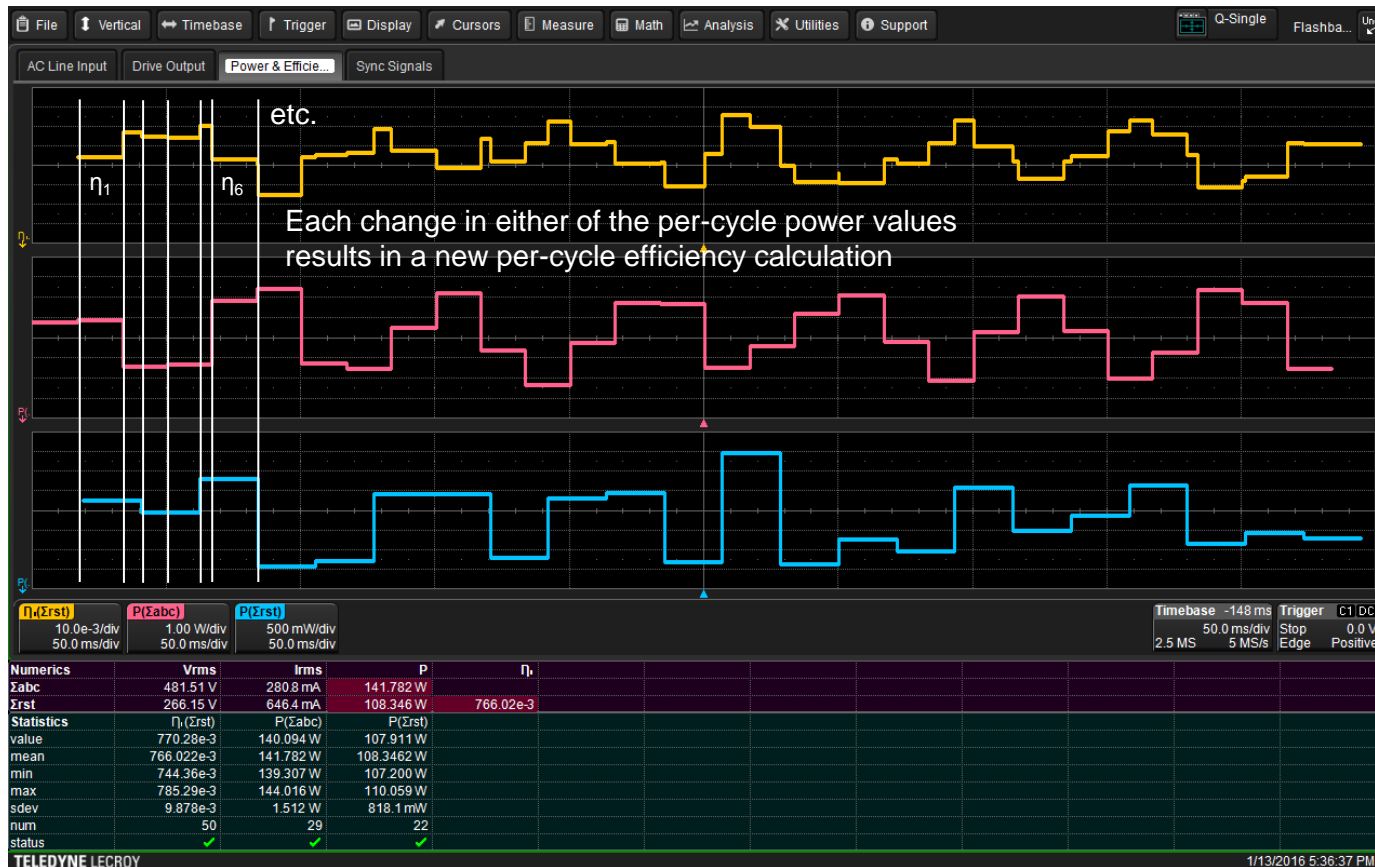
Sync signals no longer shown...but otherwise the same as the previous slide



**Note:** Waveform trace thickness has been enhanced in this image to improve viewing on a projector

# Per-cycle Efficiency, Power( $\Sigma ABC$ ) and Power( $\Sigma RST$ ) Waveforms

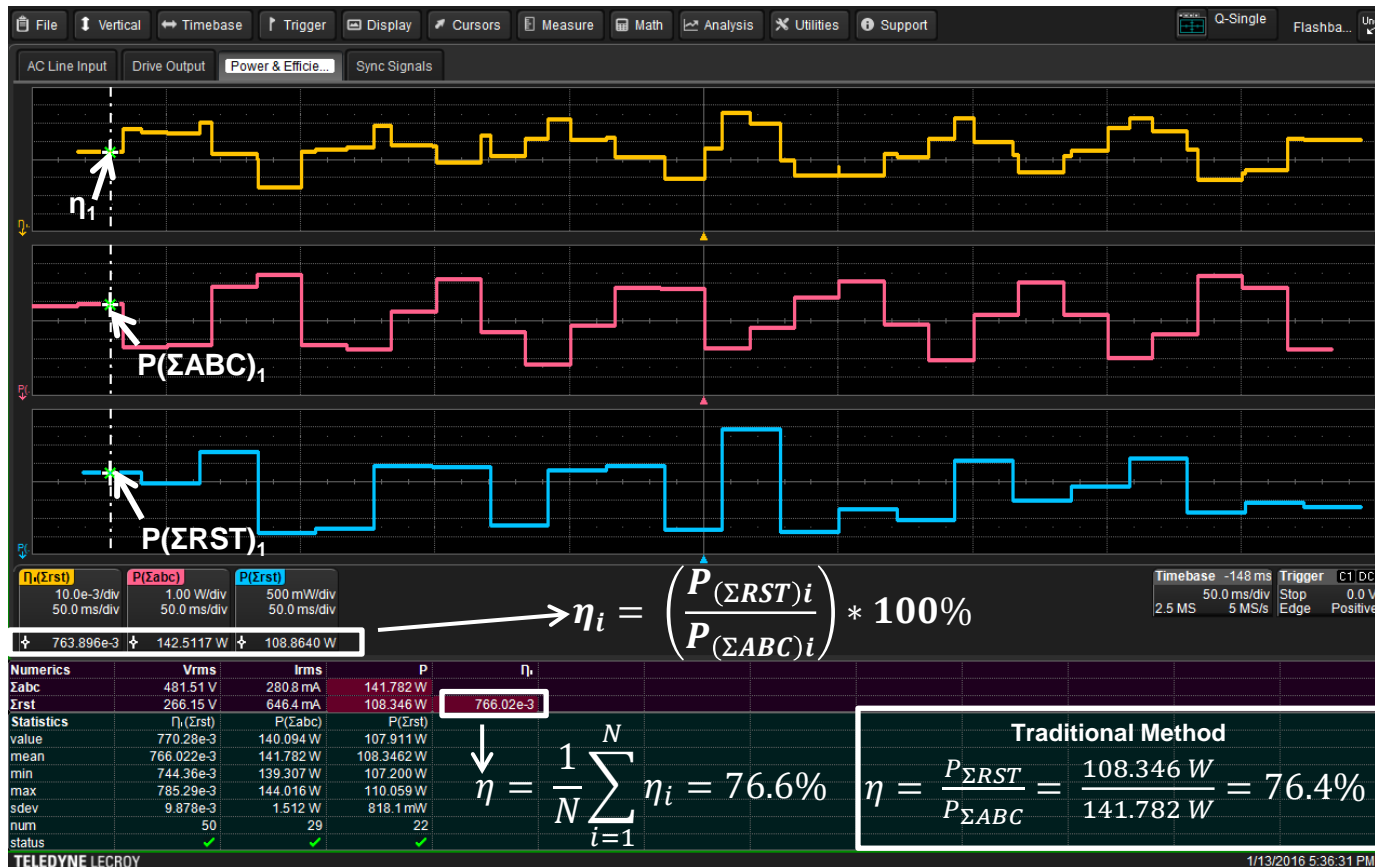
*Per-cycle efficiency is calculated anew for every new AC Input or Drive Output Sync period*



**Note:** Waveform trace thickness has been enhanced in this image to improve viewing on a projector

# Per-cycle Efficiency, Power( $\Sigma ABC$ ) and Power( $\Sigma RST$ ) Waveforms

Efficiency calculation detailed comparison



50 Efficiency Calculations

29 Line Input Power Calculations

22 Drive Output Power Calculations

**Note:** Waveform trace thickness has been enhanced in this image to improve viewing on a projector

# Static Efficiency Analysis

## Summary of methods and results

- The two methods and instruments used for calculating efficiency during a static (steady-state) operating condition achieve substantially the same result
  - Within 0.5% to 1% of each other for power
  - Slightly worse for efficiency (root sum of squares)
  - Primary difference in results is due to use of probes with the one solution, which results in some small, additional accuracy error
  - Above correlation was confirmed during numerous in-house tests and field beta tests
- To the best of my knowledge, technical test standards support use of either calculation
  1.  $\eta = \frac{1}{N} \sum_{i=1}^N \eta_i$  with  $\eta_i = \left( \frac{P_{(\Sigma RST)i}}{P_{(\Sigma ABC)i}} \right) * 100\%$
  2.  $\eta = \frac{P_2}{P_1}$  with  $P_2$  and  $P_1$  the mean output and input power values for the acquisition period

# Example and Detailed Comparison: 480V Variable Frequency Drive Input-Output **Dynamic** Efficiency Analysis

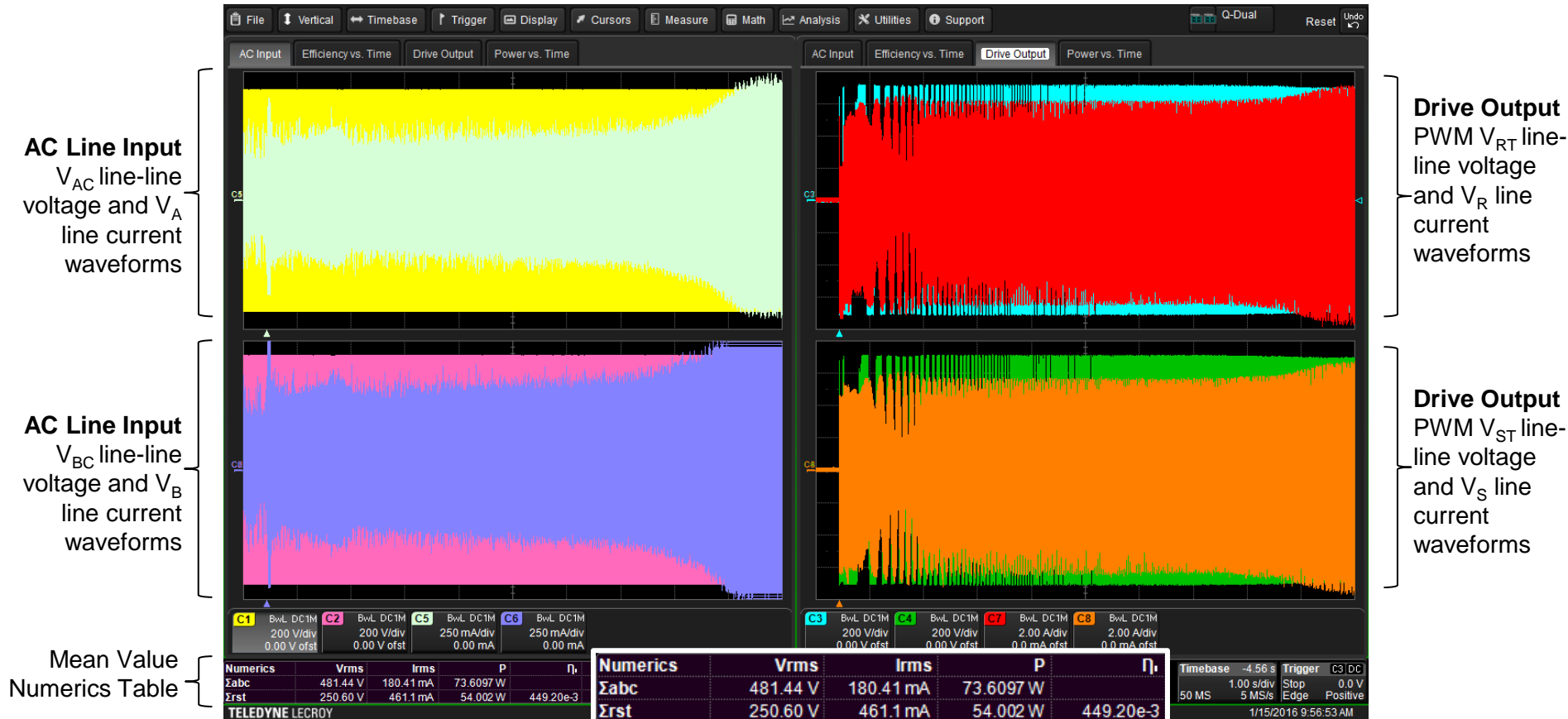
This examples demonstrates the approach taken to measure per-cycle power and efficiency during a “dynamic” operating condition when the input and output power frequencies are not the same. This example shows the use of the Teledyne LeCroy instrument, and we are seeking input from industry on the measurement methodology we employed.



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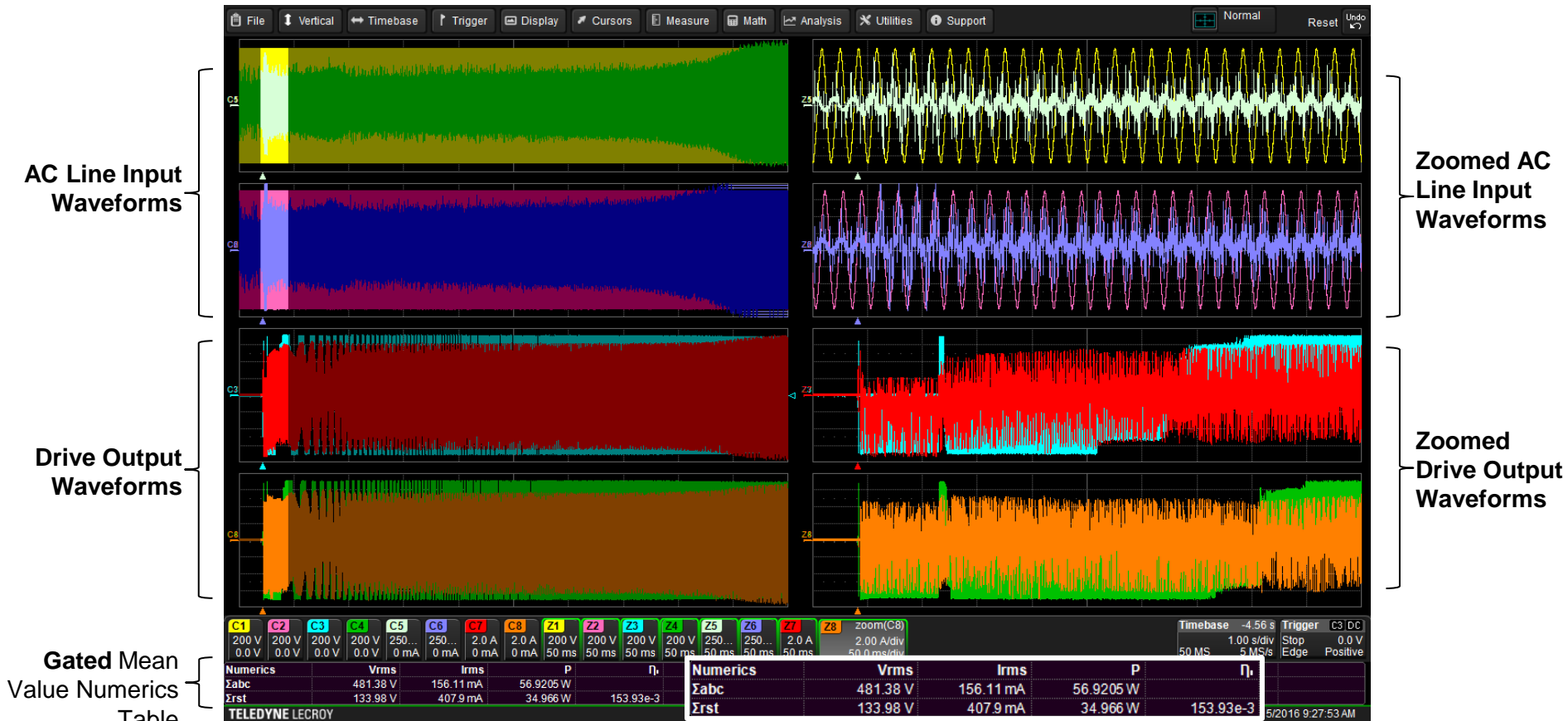
# Dynamic Power and Efficiency Analysis – AC Input to Drive Output

10s acquisition from motor startup at no-load to some applied load – 44.92% Mean Efficiency



# Acquisition Detail – Initial Drive Startup

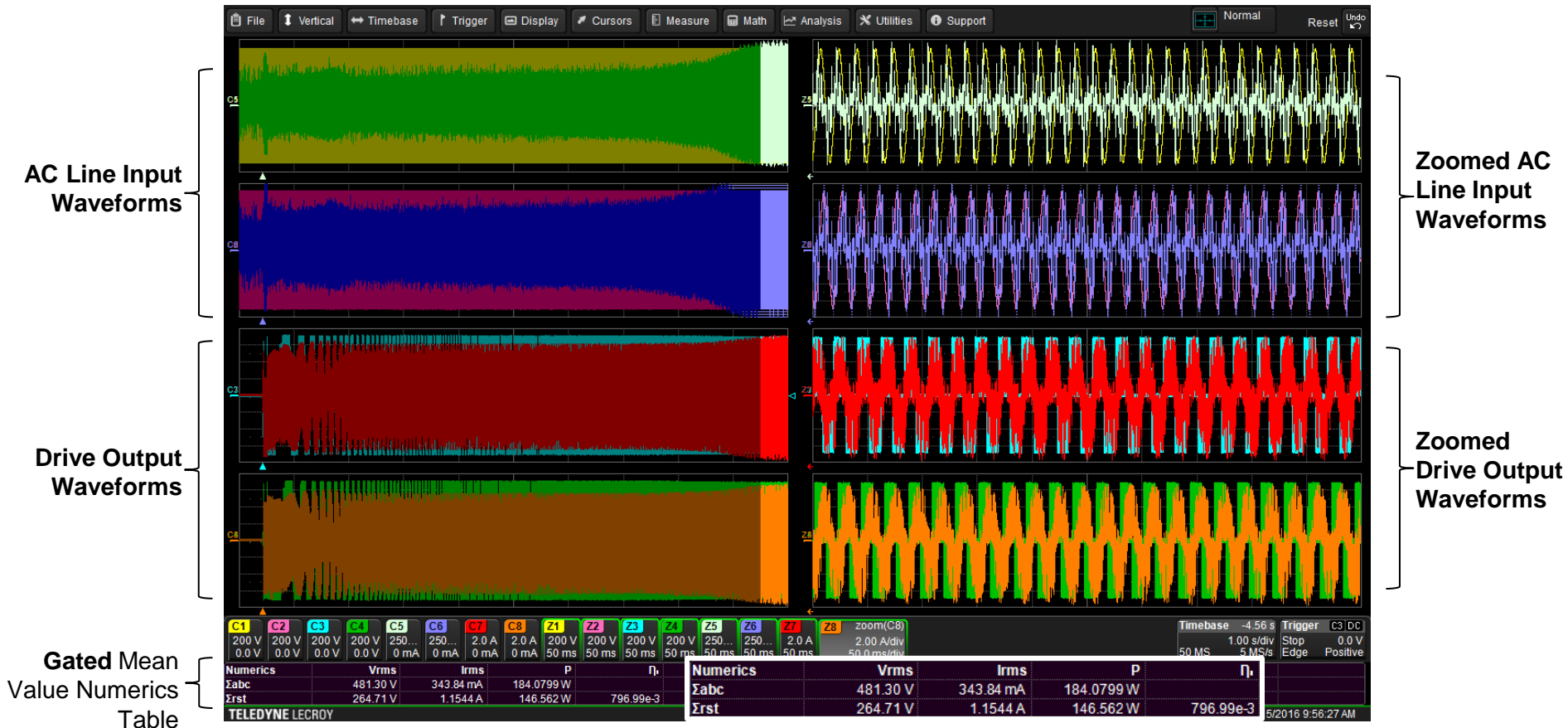
*Zoomed area indicates measurement gate – 15.39% mean Efficiency during this time interval*





# Acquisition Detail – End of Acquisition

*Zoomed area indicates measurement gate – 79.7% mean Efficiency during this time interval*



# Acquisition Detail + Per-cycle Calculated Waveforms

*All waveforms are shown here...but only a portion will be shown in the next few slides...*

AC Line Input Waveforms



Efficiency vs. Time  
Per-cycle Waveform

Drive Output Waveforms

Power vs. Time  
Per-cycle Waveform

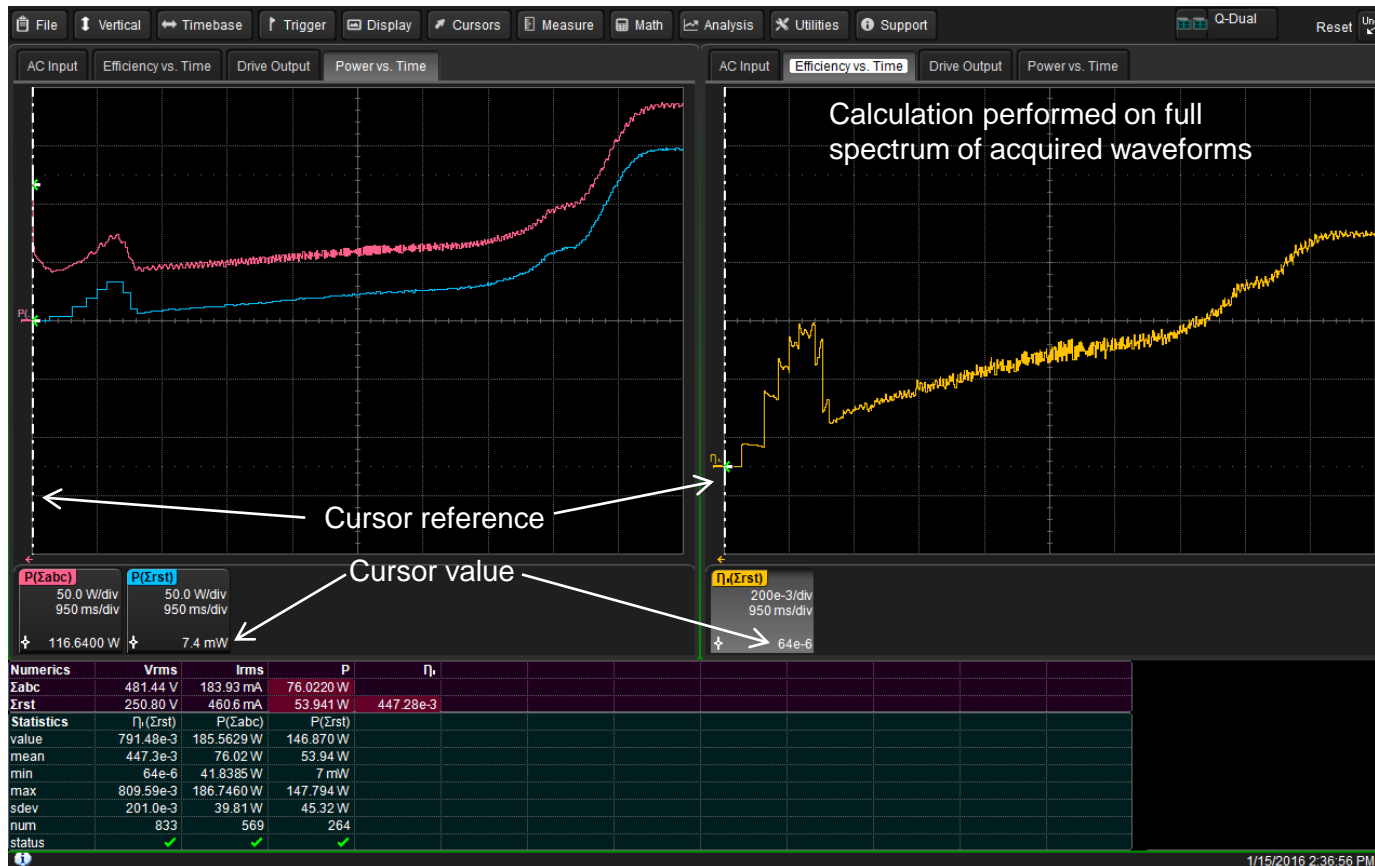
Mean Value  
Numerics Table



# Power and Efficiency Per-cycle Waveforms (Full Spectrum)

Startup to end of acquisition – 0.064% Efficiency at startup to ~80% at end (with load)

Power(ABC)  
and  
Power(RST)  
vs. Time  
Per-cycle  
Waveforms

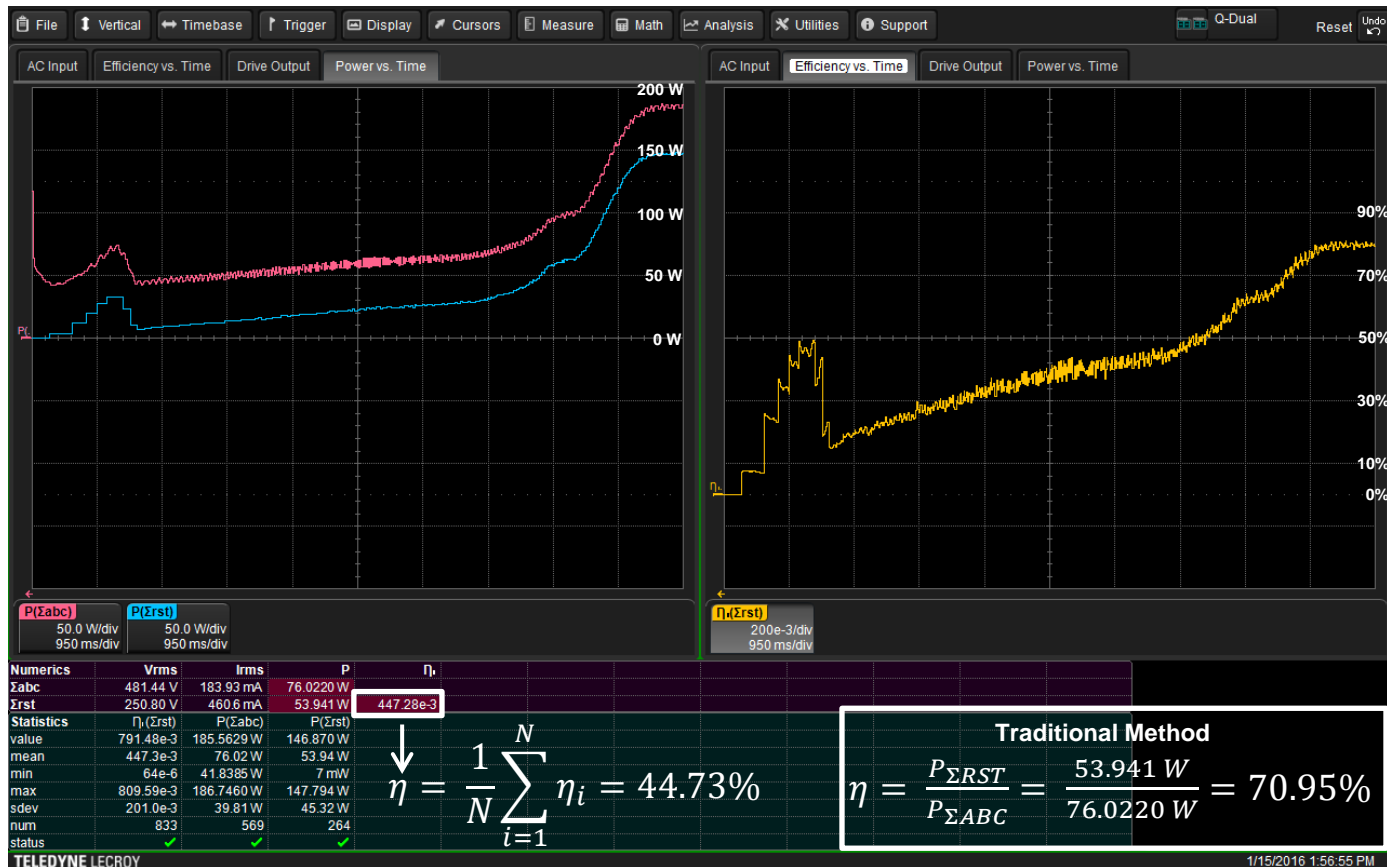


Efficiency vs.  
Time  
Per-cycle  
Waveform

# Power and Efficiency Per-cycle Waveforms (Full Spectrum)

Startup to end of acquisition – 0.064% efficiency at startup to ~80% at end (with load)

Power(ABC)  
and  
Power(RST)  
vs. Time  
Per-cycle  
Waveforms



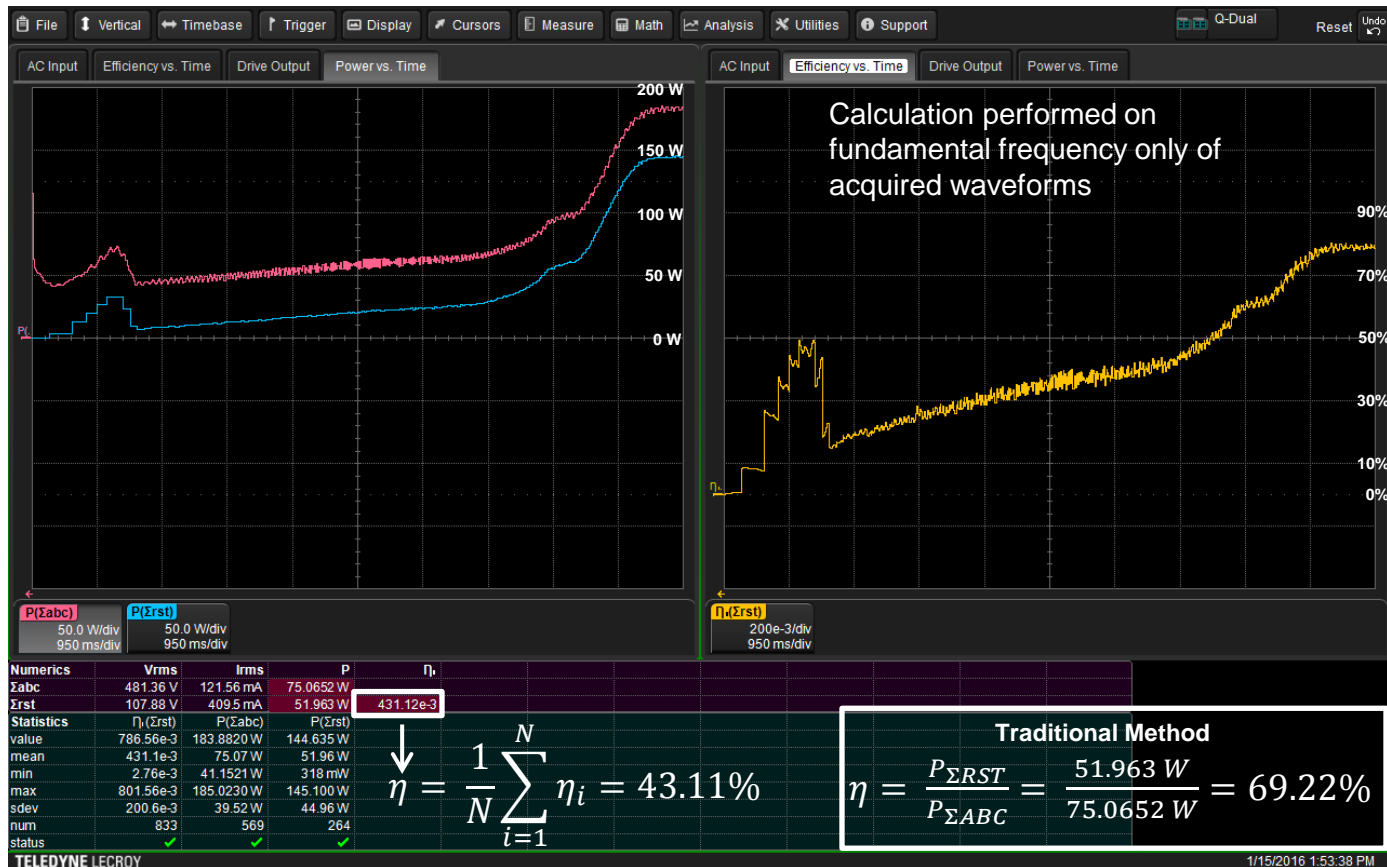
Efficiency vs.  
Time  
Per-cycle  
Waveform



# Power and Efficiency Per-cycle Waveforms (Fundamental Only)

Startup to end of acquisition – 0.276% Efficiency at startup to ~80% at end (with load)

Power(ABC)  
and  
Power(RST)  
vs. Time  
Per-cycle  
Waveforms



Efficiency vs.  
Time  
Per-cycle  
Waveform

$$\eta = \frac{1}{N} \sum_{i=1}^N \eta_i = 43.11\%$$

Traditional Method

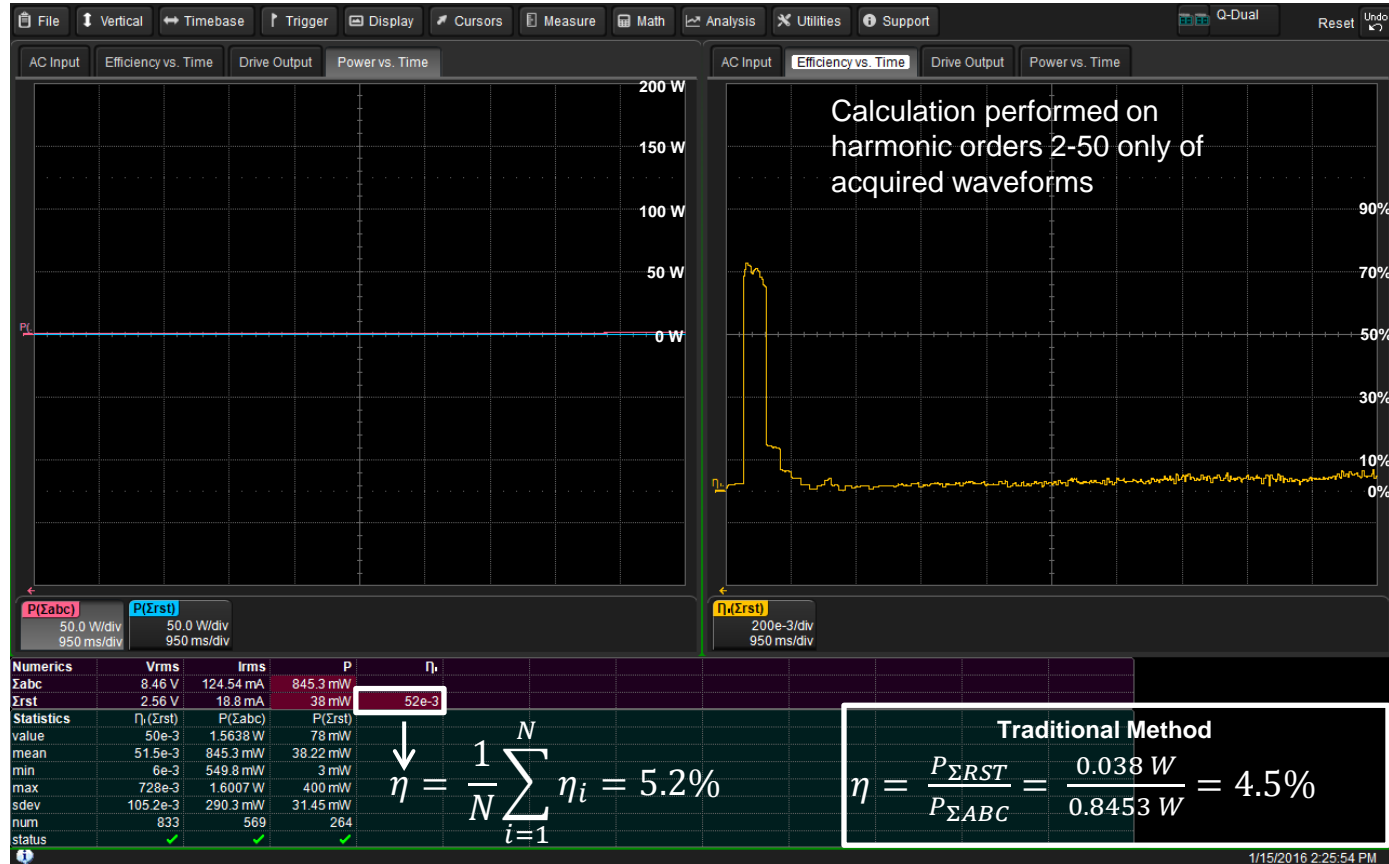
$$\eta = \frac{P_{\Sigma RST}}{P_{\Sigma ABC}} = \frac{51.963 \text{ W}}{75.0652 \text{ W}} = 69.22\%$$



# Power and Efficiency Per-cycle Waveforms (Harmonic Orders 2-50)

Startup to end of acquisition – 0.6% Efficiency at startup to 72.8% during startup

Power(ABC)  
and  
Power(RST)  
vs. Time  
Per-cycle  
Waveforms

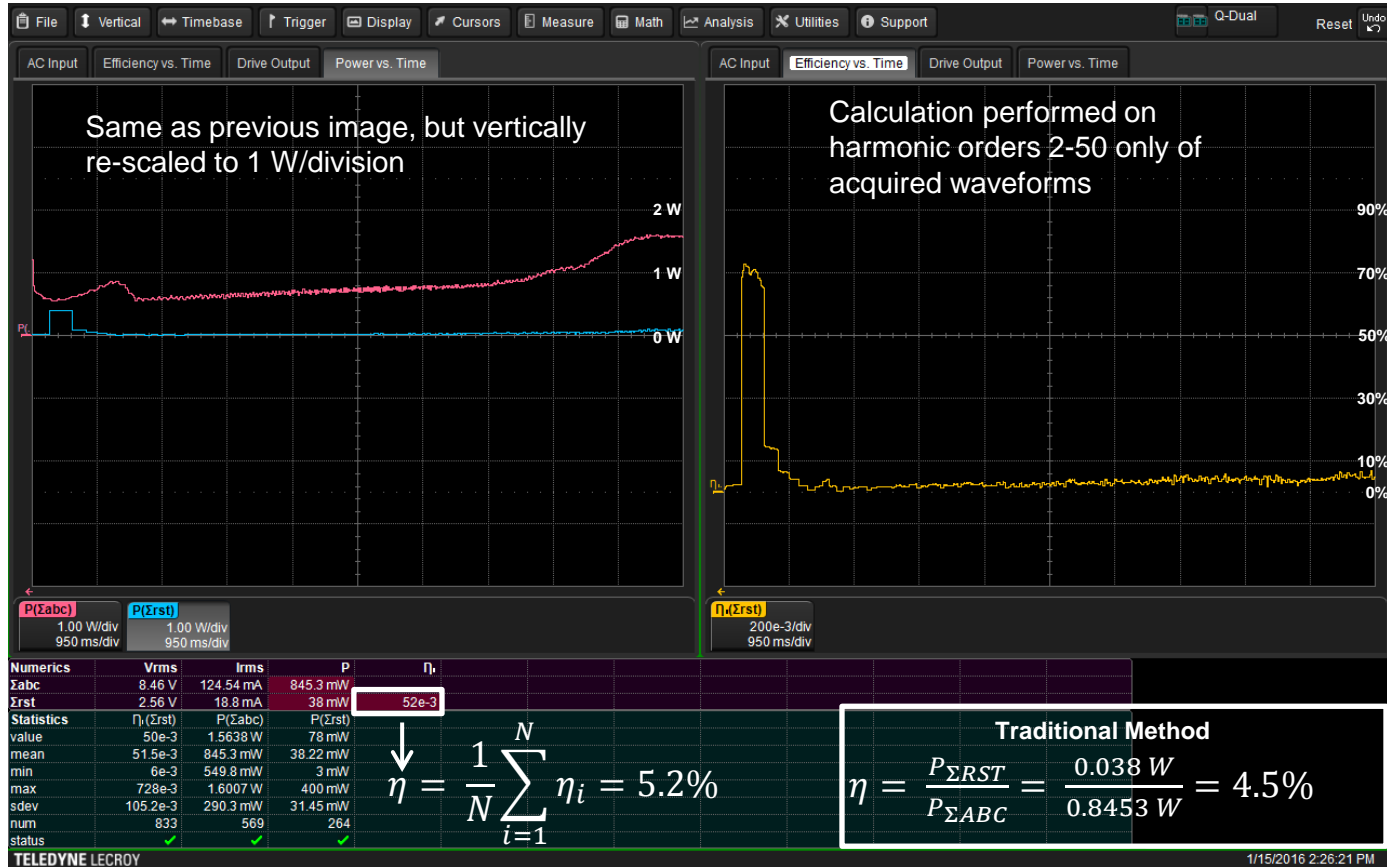


Efficiency vs.  
Time  
Per-cycle  
Waveform

# Power and Efficiency Per-cycle Waveforms (Harmonic Orders 2-50)

Startup to end of acquisition – 0.6% Efficiency at startup to 72.8% during startup

Power(ABC)  
and  
Power(RST)  
vs. Time  
Per-cycle  
Waveforms





# Dynamic Efficiency Analysis

## *Summary of methods and results*

- Traditional methods for calculating mean efficiency from mean power values produce results in some circumstances that do not accurately reflect performance.
  - e.g., dynamic operating conditions with different input and output operating frequencies
- To the best of our knowledge, there is no technical standard that describes how efficiency should be calculated during dynamic operations when the frequencies of the inputs and outputs are different.
- We have created some new measurement capabilities for dynamic efficiency, and are proposing the described method as one possible method for dynamic efficiency calculation.
- We are seeking industry partners to learn from and work with.
- Contact:
  - Ken Johnson, Director of Marketing, Product Architect [ken.johnson@teledynelecroy.com](mailto:ken.johnson@teledynelecroy.com)

# Questions or Comments?



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