Computation of Time Domain Impedance Profile from S-Parameters: Challenges and Methods

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Abstract

When computing time-domain impedance profile from measured S-parameters, we face two types of problems. First is caused by low quality of sampled S-parameters, such as insufficient resolution, band-limiting, noise etc. These limitations create certain numerical challenges, such as response aliasing, ringing and non-causality. The computed impedance becomes sensitive to termination conditions set up at the far end of the connector. The second problem is the fact that even with the ideal data, the computed impedance profile does not show the correct characteristic impedance of each section of the connector, because multiple reflection mask the ‘true’ value of the characteristic impedance of the distant sections. In the paper, we address both problems: first, by post-processing inverse discrete Fourier transform (IDFT) results, or by vector fitting. Second is addressed by modeling the device under test (DUT) as a cascade connection of multiple small transmission lines and applying time domain gating and peeling, to calculate the length and impedance of each transmission line, so that the impedance profile of the combined cascaded system is equivalent to that of the DUT.

Biography

Dr. Vladimir Dmitriev-Zdorov is a Principal Engineer in the System Design Division at Mentor Graphics Corporation. He has developed a number of advanced models and novel simulation methods used in the company’s products. Currently his work includes development of the efficient methods of circuit/system simulation in time and frequency domain, transformation and analysis of the multi-port systems, statistical & time domain analysis of SERDES links. He received Ph. D and D. Sc degrees (1986, 1998) based on his work on circuit and system simulation methods. The results of his work have been published in numerous papers, conference proceedings, and a monograph.

Dr. Kaviyesh Doshi received his Ph. D in electrical engineering from University of California, Santa Barbara in 2008.

He joined Teledyne LeCroy, a manufacturer of high-performance measurement instruments located in Chestnut Ridge, New York in 2008 as a Research & Development Engineer. At Teledyne LeCroy, he has been involved in the design and development of SPARQ - a time-domain reflectometry (TDR) based instrument for measuring S-parameters.

Dr. Doshi has one patent and many others applied for in the area of de-embedding and signal processing for time-domain network analysis.

Pete Pupalaikis received the B.S. degree in electrical engineering from Rutgers University, New Brunswick, New Jersey in 1988.

He has worked at Teledyne LeCroy (formerly LeCroy Corporation) located in Chestnut Ridge, New York since 1995 and currently manages integrated-circuit development, signal processing and intellectual property as Vice President of Technology Development. Prior to LeCroy, he served in the United States Army and as a consultant in embedded systems design.

Mr. Pupalaikis has thirty two patents and has published numerous papers in the area of measurement instrument design and is a member of Tau Beta Pi, Eta Kappa Nu and the IEEE signal processing, instrumentation, solid-state circuits and microwave societies. In 2013 he was elevated to IEEE Fellow for contributions to high-speed waveform digitizing instruments.
Introduction

The time-domain impedance profile of cables and connectors has remained for decades an important indicator of their characteristics such as characteristic impedance and delay, or transmission problems that include reflections caused by impedance discontinuities, attenuation, skew and some others. Historically, time-domain reflectometry (TDR) impedance first was a measurement technique. Later on, when operational frequencies advanced, the need for detailed electrical simulation necessitated full S-parameter measurements, and the connector vendors supplied either TDR results and S-parameters, or – more frequently - only S-parameters. Today, TDR impedance characteristics remain of great importance for designers, but more and more often these should be found directly from S-parameters by computations. To save processing time and disk space, the sampling of measured S-parameter data is often made barely enough to correctly represent the connector’s delay, and the upper frequency comes short to what is required by the Nyquist criteria for a given data rate. In TDR impedance computations, we deal with two times the connector delay, which makes given S-parameters even more under-sampled considering this particular purpose. The latter creates some computational challenges which we are going to address in the paper. The second issue, perhaps even of greater importance, is the fact that what we observe in the impedance plot may not be the actual characteristic impedance of the transmission line, if it is masked by a segment with different characteristics.

A commonly used method to obtain the impedance profile from S-parameters is by using inverse discrete Fourier transform (IDFT) of the return loss. This method does not take into account the multiple reflections due to impedance discontinuities. These impedance discontinuities introduce multiple reflections, which act as secondary sources. If not accounted for, these multiple reflections can give incorrect result for impedance profile. In this paper we propose an alternate way of computing the impedance profile by modeling the line as that obtained from the cascade connection of small transmission line structures. The method involves calculating the impedance for small section at a time and then undoing the effects of this small section before proceeding to calculate the impedance for the following sections. As a by-product of the algorithm, we describe the assumption involved in calculating the impedance profile by performing the IDFT of the return loss. It turns out that for situations where the characteristic impedance of the line is close to the reference impedance, the approximate method of using the IDFT does not introduce large errors and can be used. Nevertheless appropriate care needs to be taken to compute the IDFT of return loss to account for the discrete nature of the S-parameters, band-limited data and the frequency spacing of the data.

The paper is organized as follows. First, we show some basic relations between characteristic impedance, voltage transfer functions and S-parameters. Then, we consider two methods of converting frequency-domain characterization into time domain: by inverse Fourier transformation of the sampled dependence and then by rational fitting represented in a form of rational fraction expansion (RFE). The two approaches are applied to the same connector model. We show that different termination conditions on the far end may affect the initial portion of the impedance plot if the data is not perfect. In the next half we describe how the impedance profile can be recovered by modeling the line as that obtained from a cascade connection of small transmission line structures. An algorithm is provided which can be easily incorporated into a computer program.
Time-domain Impedance, S-parameters and the Voltage Transfer Function

Let us consider a simple circuit where the voltage source $E(t)$ is connected through the reference impedance $Z_0$ to the transmission line, see Figure 1. The line here is represented by its equivalent circuit enclosed into the pink box. Essentially, the model consists of two parts affecting each other through the propagation operator $W(s)$, that contains the line’s delay and possibly dispersion. The component $Z$ denotes the line’s characteristic impedance, possibly frequency dependent if the line is lossy. With zero initial conditions, a step voltage $E(t)$ produces an immediate change in the input current $i_1(t)$ and voltage $v_1(t)$. The backward current $i_{b1}$ will appear at the front end only after the wave travels twice through the propagation operator, i.e. not before two times the line’s delay. During all this period, the ratio of the input voltage to input current remains equal the line’s characteristic impedance - if it is constant - or slightly varies if the line has skin or dielectric losses [1]. When the reflected current comes back this ratio may change if the load impedance differs from the line’s characteristic impedance. If the passive load matches characteristic impedance, the backward current $i_{b1}$ is zero because $i_f = i_{z2} = -i_2$ and the sum $(i_f + i_2)$ remains zero.

Since the reference resistance and source voltage are known, the line’s impedance can be determined from the input voltage and the current defined as $i_1(t) = \left[ E(t) - v_1(t) \right] / Z_0$. The ratio becomes:
Expression (1) gives a practical way of finding an impedance profile for any linear model, not necessary the transmission line. Let’s consider S-parameters, as in Figure 2. To find an impedance profile for a certain port, we need to find the port’s input voltage and then apply (1). Let’s first define the port voltage transfer function as:

\[ K_{ij}(s) = \frac{V_i(s)}{E_j(s)} \]  

(2)

Note that (2) assumes that the voltage stimulus is applied only to port \( j \) while the voltage is measured at port \( i \), including the case \( i = j \), and that all reference resistances shown in Figure 2 are port normalizing impedances.

It is easy to prove a couple of simple relations between the port voltages found in the topology of Figure 2 and the return and insertion loss parameters. In particular, if the stimulus and the port voltage refer to the same port \( i \), we can express the return loss as:

\[ s_{ii} = 2 \left( \frac{V_i(s)}{E_i(s)} \right) - 1 \]  

(3)

If the stimulus is applied to the port \( j \) and the port voltage is measured at the port \( i \), we can express the mutual port coefficient:

\[ s_{ij} = \frac{2V_i(s)}{E_j(s)} \sqrt{\frac{r_0j}{r_0i}} \]  

(4)

From (2) and (3) it follows that:

\[ K_{ii}(s) = \frac{1}{2} (1 + s_{ii}) \]

If the voltage stimulus is a Dirac impulse (i.e. \( E_j(t) = E_0 \delta(t) \)), the port voltage becomes:

\[ V_i(s) = \frac{E_0}{2} (1 + s_{ii}) \]  

(5)

By converting (5) to the time-domain and integrating, we find the voltage response on the unit step stimulus, and then the port impedance from (1). Figure 2 implies that the S-parameters describe a single-ended model. If we want to find differential or common impedance for the mixed-mode model, we still can use the same approach, with the only difference that \( s_{ii} \) in (5) is replaced by the port differential- or common-mode return-loss, and the normalizing impedance in (1) should be doubled or halved respectively. In particular, if the differential port is made from the pair of single ended ports \((k, m)\) and these single ended ports have the same normalizing impedance \( r_0 \) (as required by the Touchstone 2.0 format specification), then (5) becomes one of the following:

\[ V_{\text{diff}}(s) = \frac{E_0}{2} \left[ 1 + \frac{1}{2} (s_{kk} - s_{km} - s_{mk} + s_{mm}) \right], \quad z_{\text{diff}}(t) = 2Z_0 \frac{v_{\text{diff}}(t)}{E(t) - v_{\text{diff}}(t)} \]  

(6)
Figure 3: Return-loss (real part) specified with different resolutions: cyan - 10 MHz, blue - 20 MHz, green - 50 MHz, red - 100 MHz.

\[ V_{\text{comm}}(s) = \frac{E_0}{2} \left[ 1 + \frac{1}{2} (s_{kk} + s_{km} + s_{mk} + s_{mm}) \right], \quad z_{\text{comm}}(t) = \frac{1}{2} Z_0 \frac{v_{\text{comm}}(t)}{E(t) - v_{\text{comm}}(t)} \] (7)

What is different if some ports, for example far end ports of the connector model, are not terminated by the reference impedance? In this case we have to exclude these ports by considering their actual termination conditions. Such transformation affects parameters of all ports, but can be easily computed directly in frequency domain for any sampled data. To mitigate the effect of band limiting, it is reasonable to choose the stepwise stimulus with a finite rise time, which is equivalent to high frequency filtering. To do that, time domain integration of the voltage Dirac response is slightly modified and represented by two integrals, one for the linear ramp and the other for the constant part of the stimulus.

**Finding the Time-domain Response From Frequency Characteristics**

We have two ways of finding time-domain responses from frequency characteristics. One is an IDFT and the other is complex pole fitting [2] with direct time-domain computation. In the latter approach, a sample frequency-domain function is “fitted” and represented in a form of RFE, where coefficients and poles of the summands are real or form complex conjugate pairs:

\[ H(s) = H_\infty + \sum_{m=1}^{M} \frac{A_m}{1 + \frac{s}{\alpha_m}} \] (8)

From (8), the time-domain response can be found directly. For example, unit step response becomes:

\[ a(t) = \left[ H_0 - \sum_{m=1}^{M} A_m e^{-\alpha_m t} \right] \text{ for } t \geq 0 \] (9)

where \( H_0 = H_\infty + \sum_{m=1}^{M} A_m \). We are going to illustrate the two methods in parallel and show how data sampling affects the accuracy of time responses. We take a typical 4-port connector model, with about 3.5
Figure 4: Time-domain Responses Obtained by Various Methods:
cyan - 10 MHz, blue - 20 MHz, green - 50 MHz, red - 100 MHz.

Figure 5: Insertion Loss
cyan - 10 MHz, blue - 20 MHz, green - 50 MHz, red - 100 MHz.

Figure 6: Step Responses
cyan - 10 MHz, blue - 20 MHz, green - 50 MHz, red - 100 MHz.
ns propagation delay and non-ideal port matching. Since both single ended and mixed mode impedances depend on return loss characteristics (5-7), we investigate these first.

As we see from Figure 3, the most complicated portion of the return loss is below 2 GHz, where attenuation is not high and the signal reflected from the far end is visible. Because of 7 ns round-trip delay, the curves here aren’t smooth and suffer most from under-sampling. Figure 4 illustrates how the sampling affects time domain responses. With increased sampling step, the response found by the IDFT becomes shorter (in inverse proportion to frequency resolution) and reaches the wrong steady state. The latter is caused by aliasing: insufficient sampling can make some portion of the impulse response to appear at negative times [3]. When finding the step response, time domain integration does not include this portion therefore large time behavior is wrong. With RFE, the approximation is always causal, the duration of the response is not limited, and the steady state not affected: even seemingly wrong red waveform eventually gets to the right level even with totally incorrect large time behavior. At the same time, RFE can cope with moderate under-sampling better than IDFT because it can guess missing data between given points: the green waveform (50 MHz sampling) on the left seems more accurate than it is on the right. If however the data sampling is very sparse (e.g. 100 MHz), the RFE algorithm has more freedom to provide its own, different behavior between the points, and it has to, since any smooth interpolation does not appear causal.

Unlike return-loss, insertion loss parameters often can be represented as a product of the delay operator and smooth dependence: $S_{12}(s) = e^{-sTF(s)}$. Both IDFT and RFE make use of this possibility. The delay is extracted in frequency domain, transformation or fitting is applied to the smooth function, then the delay is added to the time domain response. Because of that, even for 100 MHz sampling all methods give us sufficiently accurate time response.

With the sampling step reaching 200 GHz, delay extraction becomes impossible since it gives less than 2 samples per $1/T$ [Hz] and the trajectory in Figure 5(c) goes counterclockwise. This results in very incorrect time domain solution, as the one nearly flat in Figure 6(b).

What is Different about TDR Impedance Computations?

Here we should mention the following:

1. The input excitation is a ramped step or pulse. The ramp size should fit the cut off frequency of the response in frequency domain, to suppress ringing.
2. The expressions in (5-7) become a combination of single-ended parameters. Due to increased complexity, the originally specified sampling may not be sufficient. For RFE, resolution is not an issue as we can combine the summands in (8) found for S-matrix components.

3. If the termination at the far end differs from the normalizing impedance, we get additional complications.

The latter includes the following. First, the reflections from far end become considerable and the front end dependencies acquire more complexity, (e.g. as in Figure 7(b) and Figure 7(c)). In the case of grounded / non-connected ports, they contain the delay, but as can be seen from the phase plot Figure 7(c), the delay operator is not any more a common multiplier. Time responses coincide up to 7 ns, two times the delay, then diverge. Since this delay is two times the connector’s delay, the existing resolution may not be sufficient for it, even though it was good enough for the insertion loss. Since we cannot represent delay operator as a multiplier, we cannot get rid of it in order to make the dependence smooth and simple. Therefore the IDFT is sensitive to the resolution, and fitting requires far more poles/residues than it would take with far end ports terminated.

The responses shown in Figure 7 were found from the data defined with very fine sampling of 2 MHz. More often, we deal with much coarser resolution, affecting time-domain responses Figure 8.

**How Can We Improve Impedance Computations by the IDFT?**

The root of the problem is computation of the Dirac impulse response from the voltage transfer function by the IDFT. An example of such impulse response is shown in Figure 9. The plot is already unwrapped: the portion to the left corresponds to negative time, to the right – positive time. The time boundaries are defined by the sampling interval of S-parameters (10 MHz).

As we see, the responses reveal non-causality. They all manifest some behavior between time -10 ns to 0. In addition, the green and red curves show some consistent behavior at the far left (between -50 ns to -30 ns). The latter seems to be the effect of aliasing caused by insufficient resolution. The former however is not caused by aliasing. This could be either due to band bounding, or just because of a general type of non-causality of the measured data which is always present to some degree. Aliasing of this type can
Figure 9: IDFT of the Differential Voltage Transfer Function
Blue is 50 Ω termination at far end. Red is grounded termination. Green is open termination.

(a) Aliased Portion is Appended to End
(b) Resulting Response
(c) Remaining Negative Time Portion is Folded Around Zero and Placed on Right Side

Figure 10: Process of Removing Aliasing

(a) 2 MHz Sampling
(b) 50 MHz Sampling
(c) 70 MHz Sampling

Figure 11: IDFT with Post-processing with Different Sample Frequency Resolution
be eliminated by appending the far left portion of the response to the right end, as shown in Figure 10(a). The separation point is defined by finding the flat portion of the integral of the impulse response. After the aliased segment is appended to the right we have the result shown in Figure 10(b). The remaining non-causality is removed by folding and adding the negative portion to the existing positive time response as shown in Figure 10(c).

Even though such procedure does not make impedance plots ideal, it provides considerable improvements. Compared to Figure 8, even with 70 MHz sampling the responses are not totally wrong, and with 50 MHz sampling they are improved. In all considered cases, the impedance plot with a properly terminated far end is most accurate and the difference between the plots with different termination conditions is a good indicator of the algorithm consistency.

### How Can We Improve Impedance Computations if S-parameters are Represented by RFE?

Let’s now assume that S-parameters are defined by an RFE: each matrix component is represented as a sum with common or individual set of poles. The RFE is a complete and causal representation of a linear system, which makes it advantageous. Finite resolution and band bounding is not an issue for an RFE, as it
can be re-sampled with arbitrary fine step up to arbitrary large frequency. However, as we understand, the
RFE was created at some point from measured or simulated data which could be under-sampled or have
errors. To a certain extent, the RFE may improve data quality, but if it was non-causal or poorly sampled, it
may add an “unintended” behavior, compare Figure 12(a) and Figure 12(b). Once created, the RFE cannot
be “improved” by interpolation because it is already an analytical representation. Fitting/building an RFE
should preferably be made to the complete non-reduced matrix, where the complexity of every component
in term of poles/residues is typically less than of reduced matrix or the voltage transfer function.

To find an impedance profile, we need to find the port voltage in time domain. As follows from (5-7),
voltage transfer functions in RFE form can be found directly from an RFE representation of S-parameters,
but only if all other ports are terminated by their normalizing impedances. To evaluate the error of TDR
computations, we’d like to have them done with different termination conditions, as we did with the IDFT.
To make it possible, we either need to re-sample the RFE and transform S-matrix for every frequency point,
or find the resulting transfer function in RFE form directly. These are possible scenarios:

1. Re-sampling, matrix conversion, re-fitting or IDFT
   (a) Re-sample RFE defined S-parameters with sufficiently fine resolution, to allow more complex-
       ity than can be seen for each matrix component.
   (b) Find sampled dependence of the required return loss characteristic with appropriate termination
       applied
   (c) Re-fit found dependence
   (d) Convert new fit into time response, then impedance plot
   (e) A version with the IDFT: instead of (c,d), apply the IDFT to sampled dependence

2. Finding new poles from a state-space representation (SSR)
   (a) Convert an original RFE matrix into state space equations.
   (b) Apply termination conditions, update matrices in state space representation
   (c) Convert the new SSR into a scalar RFE for the return loss parameter
   (d) Convert the RFE into the time response and impedance plot

In the first approach, we found that performing the IDFT after re-sampling the RFE by 10 MHz allows
more accurate impedance computation than with re-fitting, as the deviation between different cases does
not exceed 0.3-0.5 % versus 0.7 %, (see Figure 13(a) and Figure 13(b)). This is because precise refitting
is difficult for the transfer function, due to its higher complexity.

The second approach is based on the fact that RFE can be converted directly, without re-sampling and
refitting. It is known that RFE is equivalent to a certain state space representation in (10) where \(X \in \mathbb{C}^p\)
is a vector of state variables; \(U \in \mathbb{C}^N\) is an input and \(Y \in \mathbb{C}^N\) is an output vector:

\[
\begin{pmatrix}
\dot{x} \\
y
\end{pmatrix} =
\begin{pmatrix}
\alpha & \beta \\
\chi & \delta
\end{pmatrix}
\begin{pmatrix}
x \\
u
\end{pmatrix}
\text{ or }
\begin{pmatrix}
0 \\
Y
\end{pmatrix} =
\begin{pmatrix}
\alpha - sI & \beta \\
\chi & \delta
\end{pmatrix}
\begin{pmatrix}
X \\
U
\end{pmatrix}
\]

(10)

We assume that \(Y\) and \(U\) have identical size and therefore input /output relationships can be described
by a square matrix , as in case of S-parameters:
The relationships between the matrices $\alpha$, $\beta$, $\chi$, $\delta$ and poles/residues are known (see e.g. [4]) so that RFE in form (8) can be built from these matrices. The poles then become eigenvalues of $\alpha$, and the coefficients are expressed via $\chi$ and $\beta$. Conversely, if the RFE exists for all components of the transfer matrix $H(s)$, the matrices $\alpha$, $\beta$, $\chi$ and $\delta$ can be easily found, too.

Our task is to find the RFE of the reduced transfer matrix, in assumption that some ports of the S-parameter model are eliminated because of certain termination conditions. Let the original equations be:

$$
\begin{pmatrix}
S_{11} & S_{12} \\
S_{21} & S_{22}
\end{pmatrix}
\begin{pmatrix}
A_1 \\
A_2
\end{pmatrix} =
\begin{pmatrix}
B_1 \\
B_2
\end{pmatrix}
$$

(12)

The termination conditions applied to the ports collected in the second sub-vector:

$$
A_2 = \Re B_2
$$

(13)

The components of the diagonal matrix $\Re$ are reflection coefficients of the terminator, $\rho_i = \frac{z_i - z_{0i}}{z_i + z_{0i}}$, which always exist if the termination is passive; $z_{0i}, z_i$ are port normalizing and terminating impedances respectively. Since the reflected wave $B$ is an output, and the incident wave $A$ is an input, we can rewrite (10) as:

$$
\begin{pmatrix}
0 \\
Y_1 \\
Y_2
\end{pmatrix} =
\begin{pmatrix}
\alpha - sI & \beta_1 & \beta_2 \\
\chi_1 & \delta_{11} & \delta_{12} \\
\chi_2 & \delta_{21} & \delta_{22}
\end{pmatrix}
\begin{pmatrix}
X \\
U_1 \\
U_2
\end{pmatrix}
$$

(14)

In (14), $U_2 = \Re Y_2$. From the last equation: $Y_2 = \chi_2 X + \delta_{21} U_1 + \delta_{22} \Re Y_2$ we get $Y_2 = (I_{22} - \delta_{22} \Re)^{-1} \chi_2 X + \Re (I_{22} - \delta_{22} \Re)^{-1} \delta_{21} U_1$. Then, (14) reduces to:

$$
\begin{pmatrix}
0 \\
Y_1
\end{pmatrix} =
\begin{pmatrix}
(\alpha - sI) + \beta_2 \Re (I_{22} - \delta_{22} \Re)^{-1} \chi_2 & \beta_1 + \beta_2 \Re (I_{22} - \delta_{22} \Re)^{-1} \delta_{21} \\
\chi_1 + \delta_{12} \Re (I_{22} - \delta_{22} \Re)^{-1} \chi_2 & \delta_{11} + \delta_{12} \Re (I_{22} - \delta_{22} \Re)^{-1} \delta_{21}
\end{pmatrix}
\begin{pmatrix}
X \\
U_1
\end{pmatrix}
$$

(15)

If the termination impedance is not frequency dependent (e.g. resistive, open or grounded) then only the upper left block changes with frequency, and (15) and (10) have similar structure. Finally, from (15) we get the new state-equation matrices and build a new RFE that describes the reduced model with some ports terminated, grounded or disconnected. Then from (5-7), we find the differential voltage transfer function, voltage waveform and an impedance profile. With grounded or disconnected far end, the converted RFE contains much more poles (real ones and complex conjugate pairs), 404 total, than in case of the matched termination (340). The TDR impedance plot is shown in Figure 13(c). Here we observe better agreement between the curves, with a difference of only about 0.1%.

In the above transformations, we assumed that the RFE represents S-parameters with no error, therefore all discrepancies in TDR impedance plots were caused by the transformations. With non-ideal RFE, TDR impedance plots found with different termination conditions would manifest a certain non-removable difference. The root of this difference is inaccurate representation of the delay operator ‘embedded’ into the voltage transfer function. This makes a small part of the reflected power to appear as ‘non-reflected’ and start earlier in time than it should.
Impedance profile using cascaded Transmission line model for the device under test (DUT)

Another approach to calculating the impedance profile is by modeling the DUT as a cascade connection of multiple transmission line structures. This method was briefly described in the de-embedding tutorial presented at DesignCon [5]. In this paper we will describe the method in detail. Instead of Figure 1 we will use Figure 14 to model a lossless transmission line. This model is more conducive from S-parameters and power waves point of view instead of currents and voltages. Such a lossless, ideal transmission line can be described by a characteristic impedance $Z_c$ and a propagation time $T$. In Figure 14, the characteristic impedance is incorporated in $z$ as the match to the reference impedance $Z_0$ (usually $50 \, \Omega$) as:

$$
\rho = \frac{Z_c - Z_0}{Z_c + Z_0}.
$$

(16)

For the ideal, lossless transmission line with no frequency dependent delay, the delay of the line $T$ is incorporated in $z$ as:

$$
z = e^{j2\pi fT}.
$$

(17)

The transmission line model shown in Figure 14 can be thought of as composed of cascade connection of three different two-port networks as shown in Figure 15. Describing from the left, the first dotted line box corresponds to a network that models the impedance change, i.e. the wave will see a change of impedance from $Z_0$ to $Z_c$ and this is reflected by an $S_{11}$ of value $\rho$. Due to this impedance change, part of the wave will be reflected and rest of the wave will be transmitted through. The next two-port section is shown by the solid box. Here the characteristic impedance is $Z_c$ and the wave does not see any impedance change. As a result the wave gets transmitted through. The through wave is delayed by the $S_{21} = z^{-1}$. The third section is again an impedance transformer what transforms the impedance the wave sees - from $Z_c$ to $Z_0$. The combined effects of all the impedance transformations can be described by the S-parameters of the transmission line model of Figure 14:

$$
\begin{pmatrix}
\rho \cdot (1 - z^{-2}) & (1 - \rho^2) \cdot z^{-1} \\
(1 - \rho^2) \cdot z^{-1} & \rho \cdot (1 - z^{-2})
\end{pmatrix}
\frac{1}{1 - z^{-2} \cdot \rho^2}.
$$

(18)

Some more reality can be added by incorporating a loss and group delay that is a function of frequency $G(f)$ and $D(f)$, respectively. For the line that is lossy or contains frequency dependent loss or delay, we use the following equation:
\[ z = G(f) e^{j2\pi f(T + D(f))} \] (19)

The model shown in Figure 14 forms the “basic unit” i.e. in the impedance profile model, we consider the line to consist of several such small transmission line sections cascaded together. This model is shown in Figure 16. For this model, we will first generate the equations that allow the determination of forward and reverse waves at various times and locations along the line, knowing the \( \rho \) and \( z \) of each section. Then we will address the inverse problem of determining the \( \rho \) and \( z \) for each individual section from the system S-parameters.

To simplify things, we start by simplifying the interface between two transmission line sections:

**The Transmission Line Interface**

Consider the interface between two transmission line sections shown in Figure 14. First, we want the equivalent S-parameters of the interface. Second, we will want to convert waves incident on the interface to waves present in the middle of the interface. The equivalent S-parameters of the interface can be computed by applying nodal analysis to the network shown in Figure 14. A detailed description of the nodal analysis method is provided in [5]. Here we provide the main equations to compute the S-parameters. We start by writing the nodal equations:
\[ A^{-1} x = v \] (23)

where the first column in \( v \) represents the waves present at all of the nodes defined in \( v \) when the network is driven only with unity incident waves from the left and the second column in \( v \) represents the waves present at all of the nodes defined in \( v \) when the network is driven only with unity incident waves from the right.

Therefore, the equivalent S-parameters of this network can be written as:

\[ S_{\text{interface}} = \begin{pmatrix} (A^{-1}x)_{1,0} & (A^{-1}x)_{1,1} \\ (A^{-1}x)_{4,0} & (A^{-1}x)_{4,1} \end{pmatrix} = \begin{pmatrix} \frac{\rho_{m-1} - \rho_m}{\rho_m \rho_{m-1} - \rho_m - \rho_{m-1} - 1} & \frac{\rho_{m-1} \rho_m - \rho_m - \rho_{m-1} - 1}{\rho_m \rho_{m-1} - \rho_m - \rho_{m-1}} \\ \frac{\rho_{m-1} + \rho_m - \rho_{m-1} - 1}{\rho_m \rho_{m-1} - \rho_m - \rho_{m-1}} & \frac{\rho_{m-1} \rho_m + \rho_m - \rho_{m-1} - 1}{\rho_m \rho_{m-1} - \rho_m - \rho_{m-1}} \end{pmatrix} \] (24)

Next, we obtain the values of the middle node \( f \) and \( r \) which represent the forward and reverse waves at the middle of the interface as a function of the incident waves on the network:

\[ \begin{pmatrix} f \\ r \end{pmatrix} = \begin{pmatrix} (A^{-1}x)_{2,0} & (A^{-1}x)_{2,1} \\ (A^{-1}x)_{3,0} & (A^{-1}x)_{3,1} \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} \frac{\rho_{m-1} - 1}{\rho_m (\rho_{m-1} - 1)} & \frac{\rho_{m-1} (\rho_m - 1)}{\rho_m \rho_{m-1} - \rho_m - \rho_{m-1}} \\ \frac{\rho_{m-1} \rho_m - \rho_m - \rho_{m-1} - 1}{\rho_m \rho_{m-1} - \rho_m - \rho_{m-1}} & \frac{\rho_{m-1} \rho_m + \rho_m - \rho_{m-1} - 1}{\rho_m \rho_{m-1} - \rho_m - \rho_{m-1}} \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \] (25)
Before continuing, let’s make some further simplifications. We know that \( \rho_m = \frac{Z_m - Z_0}{Z_m + Z_0} \) and therefore, substituting into (24), the S-parameters of the network become:

\[
P_m = S_{\text{interface}_m} = \begin{pmatrix} \rho'_m & 1 - \rho'_m \\ 1 + \rho'_m & -\rho'_m \end{pmatrix}
\]  

(26)

Where:

\[
\rho'_m = \frac{Z_m - Z_{m-1}}{Z_m + Z_{m-1}} = \frac{\rho_m - \rho_{m-1}}{1 - \rho_m \rho_{m-1}}
\]  

(27)

(27) is intuitive because it states that the S-parameters of the interface between two lines is given by the mismatch between the two lines, not the mismatch between the impedance of the lines and the reference impedance \( Z_0 \). In other words, cascaded lines that have the same characteristic impedance do not cause any reflections or degradation of the forward propagating wave at the interfaces because the lines are matched to each other (the only reflections occur getting the signal into this line in the first place when the driving impedance is 50 \( \Omega \)).

**The Solution for the Waves Along the Line**

For the moment, let’s disregard the waves at the interfaces of the lines and instead concentrate on some nodes that we can work with easily. In this spirit, we redraw the model shown in Figure 16 using the S-parameters for \( S_{\text{interface}} \) as defined in (26) to obtain a new diagram as shown in Figure 18. In Figure 18, we define \( P_m \) to be the S-parameters of the interface between sections \( m - 1 \) and \( m \). Also, the nodes \( F_{nk} \) and \( R_{nk} \) represent the waves at the outputs of the forward and reverse delay taps, respectively. Note that \( F_{nk} \) and \( R_{nk} \) don’t really exist and will need to be converted to the actual forward and reverse going waves at the transmission line section interfaces. Note also that there is a small problem at the ends of the line. The problem is that for a series of cascade lines, there is no \( \rho \) defined before the beginning of the line or after the end of the line. This is handled by defining \( \rho_{-1} \) and \( \rho_M \) as the source and terminating impedance. Usually, this is set to zero to indicate that the system starts as being driven through an impedance equal to the reference impedance \( Z_0 \) and ends in a termination of \( Z_0 \), but this is arbitrary.

Now, let’s write the equations for the \( F_{nk} \) and \( R_{nk} \). We write these by going no further from the node being defined to another defined node. We therefore get:

\[
F_m = z^{-1}P_{m-121}F_{m-1} + z^{-1}P_{m-122}R_{m-1}
\]  

(28)

\[
R_m = z^{-1}P_{m+111}F_{m+1} + z^{-1}P_{m+112}R_{m+1}
\]  

(29)

In Matrix form the nodal equations are:
\[
\begin{pmatrix}
  x_l & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & x_r
\end{pmatrix}^T = \\
\begin{pmatrix}
  1 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\
  0 & 1 & -z^{-1}P_{111} & -z^{-1}P_{112} & \cdots & 0 & 0 & 0 & 0 \\
 -z^{-1}P_{021} & -z^{-1}P_{022} & 1 & 0 & \cdots & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & 1 & \cdots & 0 & 0 & 0 \\
  \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\
  0 & 0 & 0 & 0 & \cdots & 1 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & \cdots & 0 & 1 & -z^{-1}P_{M11} & -z^{-1}P_{M12} \\
  0 & 0 & 0 & 0 & \cdots & -z^{-1}P_{M-121} & -z^{-1}P_{M-122} & 1 & 0 \\
  0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 1
\end{pmatrix}
\]

(30)

Thus, for \(M\) sections, we have a system matrix \(A\) of order \(2(M + 1)\), a nodes vector with \(2(M + 1)\) elements, where \(M + 1\) nodes are forward and \(M + 1\) nodes are reverse. To make sense of this, consider that if we have no sections, we would have two nodes, one forward and one reverse, but for a single section we have four nodes: A forward and reverse wave at each interface to the section. With two sections, we have six nodes, etc. The system matrix is thus defined as the \(2(M + 1) \times 2(M + 1)\) element identity matrix with, for \(m \in \{0 \ldots M - 1\}\), the following additions:

\[
A_{2(m+1),2m} = -z^{-1}P_{m21}
\]

(31)

\[
A_{2(m+1),2m+1} = -z^{-1}P_{m22}
\]

(32)

\[
A_{2m+1,2(m+1)} = z^{-1}P_{m+111}
\]

(33)

\[
A_{2m+1,2(m+1)+1} = z^{-1}P_{m+112}
\]

(34)

With this, we now have all of the tools necessary to solve for the waves along a transmission line modeled as described up to now. Given a sample rate \(F_s\) for the waves and number of sections \(M\) of length \(T = \frac{1}{2F_s}\) and the characteristics of each section \(m \in \{0 \ldots M - 1\}\), the values that define each section, namely \(Z_m, G_m(f)\) and \(D_m(f)\), and a frequency point \(n \in \{0 \ldots N\}\) where \(f_n = \frac{n}{N}F_s\):

1. Calculate the values of \(\rho_m\) and \(z_m\) using (16),(17) and (19).

2. Calculate the values of \(\rho'_m\) using (27).

3. Calculate the values of \(P_m\) using (24).

4. Set up the matrix \(A\) using (31),(32),(33) and (34) remembering to start with the \(2(M + 1) \times 2(M + 1)\) identity matrix.

5. Solve for \(F_m\) and \(R_m\) using (30).
6. Solve for the forward and reverse going waves at the interface using (25) where \( a_1 \) and \( a_2 \) in (25) are \( F_m \) and \( R_m \).

After solving for each frequency point, the waveforms can be solved using IDFT techniques.

Note that we have chosen here to view each section length as \( T = \frac{1}{2F_s} \). This means that for each sample taken, the waves actually traverse two sections. The reason for this is that because of how we’ve quantized things, if we look at the front of the line, we will get non-zero reverse wave samples only when we do things this way. As an example, on the first sample applied to the front of the line, we immediately see the impedance discontinuity due to the first section. But, until the impulse has gone to the boundary of the next section and back through the first section, we will not see that impedance discontinuity. Setting the length as we do allows each sample at the front of the line to see every impedance discontinuity. Having described the forward problem we now describe an algorithm to compute the \( \rho \) and \( z \) for each section of the transmission line from the two-port S-parameter measurements.

**The Calculation of the Impedance Profile**

The calculation of the impedance profile assumes first that the system being calculated at least roughly approximates a model generated by cascading multiple transmission line sections of various characteristic impedance. This is certainly true for many transmission line examples in signal integrity applications. Generally, the goal is to take one measurement at one port of the line and using this measurement infer the profile. It helps to have measurements at two ports to get a better estimate for the loss. In practice, the measurement of the impedance profile is usually the domain of TDR, but in fact it can be measured well, if not better using the vector network analyzer (VNA). Here, we will always assume that one can obtain s-parameter measurements which do not distinguish between either method of obtaining them.

Let’s start by launching an impulsive wave into a port in a transmission line. Assuming this line matches the model shown in Figure 16, what do we see happen? If we examine Figure 16, we see that at the moment the forward wave is launched at \( V_{f0} \), it immediately reflects back to node \( V_{r0} \). If an ideal direction coupler is employed, we could directly measure this reflected wave and, knowing the size of the incident wave, calculate \( \rho_0 \). If we measured the voltage at that moment in time, it would be the sum of these waves: \( V_{0,\ell=0} = V_{f0,\ell=0}(1 + \rho_0) \sqrt{Z_0} \). Again, knowing the forward wave in conjunction with the measured voltage allows the calculation of the reflection coefficient at the immediate interface \( \rho_0 \). In the second sample, we see the next forward wave along with a reflection at the interface to the next transmission line section. Remember, we have construed our model so that on each sample, the wave traverses two delay elements. Keeping to an impulsive forward wave, we see the wave that passed through the interface between the source (which is \( 1 + \rho_0 \) times our forward impulse) reflecting off the interface to the next section and back again. Note that this wave reflects off the initial interface and some of this wave turns around and becomes a forward propagating wave. As time continues things become much more complicated and the complication escalates as time goes on as waves reflect off of interfaces along the line.

It is good to look at Figure 16 to gain some insight into the complexity, but fortunately, this insight is not needed for the calculation of the impedance profile. All that is necessary is to know that the reflection coefficient due to the first interface can be calculated directly.

Assume that we have a measurement of \( S_{11} \) looking into the port of the system. \( S_{11} \) is a frequency-domain measurement that provides, for each frequency, the returned, or reverse going standing wave or phasor that comes back from the input port due to a unity incident or forward launched standing wave into the port. If we launch an impulse into this system, we already know that the impulse reflected back at time
zero is \( \rho_0 \) in size. But this is just the inverse DFT of \( S_{11} \) at time zero. Furthermore, since we are only interested in the first point, we calculate \( \rho_0 \) as:

\[
\rho_0 = \text{IDFT} \left( S_{11} \right)_0 = \frac{1}{2N} \left[ S_{110} + \text{Re} \left( S_{11N} \right) + 2 \sum_{n=1}^{N-1} \text{Re} \left( S_{11n} \right) \right]
\]  

(35)

Having now calculated \( \rho \) for the first section, we create a small transmission line section and de-embed it from the remainder of the system. The S-parameters for the ideal transmission line are given by (18) and that can be used to de-embed the small section from the measured two-port system S-parameters \( S_{\text{sys}} \). De-embedding techniques described in [5] can be used to obtain the new system \( S_{\text{sysNew}} \). For convenience the S-parameters are given below.

\[
S_L = \begin{pmatrix}
S_{L11} & S_{L12} \\
S_{L21} & S_{L22}
\end{pmatrix}
\]

(36)

\[
S = \begin{pmatrix}
S_{11} & S_{12} \\
S_{21} & S_{22}
\end{pmatrix}
\]

(37)

\[
S_R = \begin{pmatrix}
S_{L11} - S_{L12} & S_{L21}S_{12} \\
S_{L12}S_{21} & S_{22} - |S_L|^2
\end{pmatrix}
\]

(38)

Here \( S_L \) is the two-port S-parameters of the ideal transmission line section obtained from \( \rho \) and \( z \) (as shown in (18)), \( S \) is the two-port S-parameters of the measured system, and \( S_R \) is the two-port S-parameters obtained by de-embedding the ideal transmission line section from the measured system. Also, \( |S| \) is the determinant of the matrix (37) and \( |S_L| \) is the determinant of the matrix (36).

Let’s recapitulate what we know:

1. The IDFT of \( S_{11} \) of a system defines the reverse propagating waves emanating from a port after the application of an impulse forward wave stimulus.

2. While the nature of this response is very complicated, we know that the very first value at time zero is the \( \rho \) of the first transmission line section.

3. If we know the \( \rho \) of a section, and use a fixed time length for the section equal to one half of a sample period, we can construct an s-parameter model of the first incremental transmission line section. Incorporating non-ideal loss and group delay (as a function of frequency) is not addressed here, but the loss can be estimated if \( S_{21} \) measurement of the system is available.

Having knowledge of the system (i.e. the S-parameters) and the S-parameters for the first transmission line section provides a great advantage. This is because we can de-embed the first transmission line section from the system and repeat the exercise to determine the next \( \rho \) value of the second section. We can therefore iteratively arrive at the \( \rho \) values for the entire system.

Therefore, the method of generating the impedance profile can be performed by repeated calculating the reflection coefficient of the first section in the remaining system, generating an s-parameter model of a small transmission line section, de-embedding the section and doing this over and over again.
**Impedance Profile Generation Using $S_{11}$ Only**

In the last section we see how de-embedding calculates the remaining system after each $\rho$ value is calculated. While illustrative, this technique is unsatisfying for two reasons. First, it seems like a lot of work. Second, it implies knowledge of all of the S-parameters of the system. What is needed is a simpler algorithm that depends only on $S_{11}$.

The answer is available by examining (38), which is repeated here for convenience:

$$ S_R = \frac{\left( \begin{array}{cc} S_{11} - S_{L11} & S_{L21} S_{12} \\ S_{L12} S_{21} & S_{L22} |S| - S_{22} |S_L| \end{array} \right)}{S_{L22} S_{11} - |S_L|} $$  \hspace{1cm} (39)

We already know that we only needed $S_{11}$ to determine the $\rho$ of the section that we wish to de-embed. Here we see that the calculation of $S_{R11}$, the $S_{11}$ of the remainder of the system after we de-embed the small section, is dependent only on the $S_{11}$ of the original system, and $S_L$, the S-parameters of the section being de-embedded:

$$ S_{R11} = \frac{S_{11} - S_{L11}}{S_{11} S_{L22} + S_{L12} S_{21} - S_{L11} S_{L22}} $$  \hspace{1cm} (40)

Plugging in the S-parameters of the small section calculated from a given $\rho$, we obtain the new $S_{11}$:

$$ S_{11_{\text{new}}} = \frac{-S_{11} + S_{11} \rho^2 z^{-2} - \rho z^{-2} + \rho}{\rho^2 + S_{11} \rho z^{-2} - S_{11} \rho - z^{-2}} $$  \hspace{1cm} (41)

The algorithm is summarized in Figure 19. Remember that we use (41) to calculate a new value of $S_{11}$ for the remaining line after calculating a new value of $\rho$ for the impedance profile. If we did a Taylor series expansion of $S_{11_{\text{new}}}$, we get:

$$ S_{11_{\text{new}}} = \frac{S_{11}}{z^{-2}} + \left( 1 - \frac{1}{z^{-2}} \right) \left( 1 + \frac{S_{11}^2}{z^{-2}} \right) \cdot \rho + \mathcal{O} (\rho^2). $$  \hspace{1cm} (42)

If we neglect all powers of $\rho$ (implying a very small value of $\rho$ i.e. characteristic impedance has a value close to $Z_0$), then $S_{11_{\text{new}}}$ is given as:

$$ S_{11_{\text{new}}} \approx \frac{S_{11}}{z^{-2}}. $$  \hspace{1cm} (43)

(43) says that for small values of $\rho$, a good approximation of the $S_{11}$ of the remaining line is simply the $S_{11}$ advanced (as opposed to delayed) by twice the length of the small transmission line section we are using to approximate the line. This means that if we use a length that is one-half of a sample, the new $S_{11}$ is the original $S_{11}$ advanced by one sample. When we say a sample here, we mean a sample in the IDFT of $S_{11}$. This means that each value of $\rho$ can be computed from the first sample of the IDFT of $S_{11}$ which means that we can determine each value of $\rho$ by simply sampling the IDFT of $S_{11}$. It turns out that under most realistic circumstances, this is a very good and simple approximation of $\rho$:

$$ \rho \approx \text{IDFT} (S_{11}). $$
RealVector ImpedanceProfile(S_{11}, M)

Arguments:

ComplexVector S_{11} \{ 
    An \(N+1\) element vector of \(S_{11}\) for each frequency point including DC and the Nyquist rate 
}\}

int M \{ 
    The number of sections desired for the profile 
}\}

int m \in \{0, \ldots, M-1\}

\begin{align*}
    \rho_m & \leftarrow \frac{1}{2N} \left[ S_{110} + \text{Re}(S_{11N}) + 2 \sum_{n=1}^{N-1} \text{Re}(S_{11n}) \right] \\
\end{align*}

int n \in \{0, \ldots, N\}

Complex z \leftarrow e^{j\pi \frac{n}{N}}

S_{11n} \leftarrow \frac{-S_{11n} + S_{11a} \rho_m^2 z^{-2} - \rho_m z^{-2} + \rho_m}{\rho_m + S_{11a} \rho_m z^{-2} - S_{11a} \rho_m - z^{-2}}

return \rho

Figure 19: Impedance Profile Calculation Algorithm

Simulation results are described next and will compare the performance of the peeling algorithm with the IDFT approach.

**Simulation Results And Future Work**

We have considered two cases to test the validity of the proposed algorithm. For the first case, S-parameters of a cable were measured and the impedance profile was calculated from the two-port measurements. Next the cable was terminated with different terminations (open, short and 50 \( \Omega \) load) and the one-port S-parameters were measured. The impedance profile for the four different cases is shown in Figure 20. Although not shown here, the impedance profile calculated using the IDFT method also gives similar results. Since the characteristic impedance of the cable is close to the reference impedance this was expected.

The algorithm in Figure 19 shows \(M\) (number of sections) as an input argument. One question that we haven’t answered is how is the value of \(M\) chosen? Since each section of the transmission line is of length, \(T = \frac{1}{2F_s}\), it turns out the \(M\) can be chosen if the length of the DUT is known. One can obtain an estimate of the length of the DUT by observing the impedance profile obtained by the IDFT method or by calculating the delay from the through step (it has the phase (and hence group delay) information of \(S_{21}\)). Let’s say the length of the DUT estimated is less than \(L\) seconds. Then \(M\) is given by:

\[ M = \text{ceil} \left( \frac{L}{T} \right) \quad \text{(44)} \]

Since this value is not exact, the obvious question that comes to mind is what happens when we peel the network and calculate new value for \(\rho\) when the actual DUT has ended? By definition, once the DUT ends, the impedance profile will look like the characteristic impedance. Hence in this case \(\rho\) will be very close to 0. For the example showed in Figure 20, the cable length is somewhere between 2.2 ns and 2.4 ns. But we
Figure 20: Impedance profile for cable terminated with different terminations

![Impedance profile](image)

- Port 1: $Z_c = 50\,\Omega$, $T_d = 1\,\text{ns}$
- Port 2: $Z_c = 50\,\Omega$, $T_d = 500\,\text{ps}$
- Port 1: $Z_c = 150\,\Omega$, $T_d = 20\,\text{ps}$
- Port 1: $Z_c = 165\,\Omega$, $T_d = 200\,\text{ps}$

Figure 21: Simulated example - DUT

provide a value of DUT length as 2.7 ns i.e. somewhat longer than the actual length. As can be seen from the plots, for the two-port cable measurement and one-port cable with load termination measurement, the impedance of the DUT starts going towards $50\,\Omega$. Since the model is an idealized version (i.e. frequency dependent loss and group delay is not known), it may not reach $50\,\Omega$ right away. Further analysis of equations (18), (41) and (35) is required to obtain a theoretical limit and will not be addressed in this paper.

Next, a simulated example corresponding to the system shown in Figure 21 was used to generate the two-port S-parameters. Here the idea was to simulate a system with mostly $50\,\Omega$ transmission line, separated by very short high impedance lines. For such a system the impedance profile was calculated using the proposed algorithm and the results were compared with that obtained from the IDFT of the return loss.

As expected, the peeling algorithm gives accurate results as compared to the IDFT approach. The IDFT approach fails on two counts:

1. Although the DUT ends at 1.74 ns, the IDFT approach gives an impedance profile that continues well past 3.5 ns. This is because the secondary reflections due to the impedance changes act as sources and are not accounted for by the IDFT method.

2. The IDFT approach gives an incorrect value for the impedance profile around 1.52 ns. This error can be attributed to the fact that because of the IDFT approximation, powers of $\rho$ mentioned in (42) are neglected. For this particular case, $\rho$ is not as small and hence causes an error in the value of the impedance profile.

Both the above mentioned errors are avoided by using the peeling approach. There are a couple of short-
comings in the proposed algorithm that we have not addressed in the paper. First, what happens when we do not know the actual loss of the system? (41) that describes the calculation of new $S_{11}$ needs $\rho$ and $z$ from the previous section. If $G(f)$ and $D(f)$ (gain and frequency dependent delay) are unknown functions, then the $z$ used in the algorithm will be an idealization as given in (17). This will introduce errors in peeling equation (41). Second, we have not addressed the accuracy of peeling algorithm in the presence of noise. In practice, any error in calculating $\rho$ or $z$ for a section of transmission line (due to noise or idealization of $z$), would introduce an error in the new $S_{11}$ calculated using (41). This would cause the calculation of subsequent $\rho$ to be more noisy and the cycle of error propagation would continue. One proposed solution would be to use the knowledge of $S_{21}$ to refine the value of $\rho$ and/or obtain a non-ideal value for $z$. Step response due to $S_{21}$ for the system shown in Figure 21 is shown in Figure 23. The effects of multiple discontinuities in impedance and loss can be seen in this plot. The small blips that occur after the rising edge can be attributed to the multiple reflections acting as secondary sources. Using this information (or the actual $S_{21}$) in refining the peeling algorithm would certainly result in a better algorithm.
Conclusions

We have demonstrated impedance computation problems caused by insufficient resolution and band limiting of the S-parameters. Based on our research, we recommend that all data-purification procedures, including rational polynomial fitting, interpolation or extrapolation, should be applied to the raw non-converted data rather than voltage transfer function or any similar function of S-parameters. By comparing IDFT and RFE based approaches, we found that neither of them is perfect, but the best quality of solution can be achieved by combining their strengths. By using both methods, we were able to cross-verify the results and get a better judgment of the impedance profile computations.

We also provided an alternate algorithm that accounts for the multiple reflections due to impedance discontinuities. We compared the new algorithm to the well-known IDFT approach and showed for what kind of DUTs the IDFT algorithm can be used. Drawbacks of the proposed algorithm were described and a possible solution was mentioned to further improve the accuracy of the proposed algorithm.

Acknowledgments

The authors wish to thank Balamurali Bhat, Aditi Sheth and Matthew Weinstein of Teledyne LeCroy for providing the data to validate the peeling algorithm.

References


