Summary
Many physical phenomena result in waveforms with exponentially rising and/or falling edges. In such cases, the exponential time constant reveals information about the underlying process.

Introduction
It is possible to determine the time constant of an exponentially decaying signal by direct measurement using cursors or measurement parameters. Alternatively, we can verify that the waveshape is indeed exponential by logarithmically weighting the signal. If the result is linear, it indicates that the source waveform is exponential. Thus, we can read the slope of the signal directly.

Let's look at a typical application. In Figure 1 we have an exponentially falling waveform. A typical exponential process can be described by an equation of the form:

\[ V(t) = ae^{-t/\tau} + b \]

where:
- \( a \) and \( b \) are arbitrary constants
- \( \tau \) is the exponential time constant
- \( t \geq 0 \).
The time constant is defined as the time value that results in the value of the waveform falling to 37% (1/e) of its initial value. This measurement can be made in a number of ways. The first method is to use relative horizontal cursors to measure the time difference between two amplitude values with a ratio of 0.37. This is illustrated in Figure 2.

Set the upper (left-most) cursor, note the amplitude readout. Calculate 37% of that reading and move the lower (right-most) cursor to a horizontal location where the amplitude is that value. The time constant is the time between cursors. In Figure 2, the $\Delta X$ cursor reading is 1.013 µs.

A more direct technique is to use the delta time at level (dt@lvl) parameter. This measures the time between two user-set levels as shown in Figure 3. The threshold levels should have a ratio of 37%. The example in figure 3 has the upper threshold set to 95%. The lower threshold is 0.37 of this, or 35%. Detailed parameter markers show the thresholds as horizontal dashed lines. The corresponding threshold crossing times are marked with dashed vertical lines and the time difference between those lines is read out as the dt@Lvl parameter, 1.005 µs. This number is a better estimate of the time constant because the threshold can be set more accurately than the cursor positions.
Another approach is to take the natural logarithm of the exponential signal using waveform math and to measure the slope of the resulting waveforms as shown in Figure 4.

Figure 3: Measuring the time constant using the delta time at level parameter

Figure 4: Determining the time constant by taking the slope of the natural logarithm of the exponential waveform
The natural logarithm of an exponential function is a linear ramp as seen in figure 4. If we look at this mathematically:

\[
\ln [V(t)] = \ln [ae^{-t/\tau} + b]
\]

\[
\ln [(V(t) - b) / a] = -t/\tau
\]

\[
\tau = -t / \ln[(V(t)-b)/a]
\]

The time constant is the reciprocal of the slope of the ramp. In Figure 4, the slew rate parameter (P2) is used to measure the slope of the math function. The threshold levels were set to the natural logarithm of the 95% and 35% values used in figure 3. P3 is set up using parameter math to take the reciprocal of P2; this is the time constant of the original exponential 1.006 µs.

The use of the delta time at level parameter is the easiest and most accurate measurement technique but the other methods may serve as well.