Measuring Exponential Decay Slope

APPLICATION BRIEF - LAB WM416D

Waveform Math Determines Exponential Time Constants

April 17, 2012

Summary

Many physical phenomena result in waveforms with exponential rising and/or falling edges where the exponential time constant reveals information about the underlying process.

It is possible to determine the time constant of an exponentially decaying signal by direct measurement using cursors. Alternatively, we can verify that the waveshape is indeed exponential by logarithmically weighting the signal. If the result is linear it indicates that the source waveform is exponential and we can read the slope of the signal directly.

Let's look at a typical application. In Figure 1 we have an exponentially falling waveform.

A typical exponential process can be described by an equation of the form:

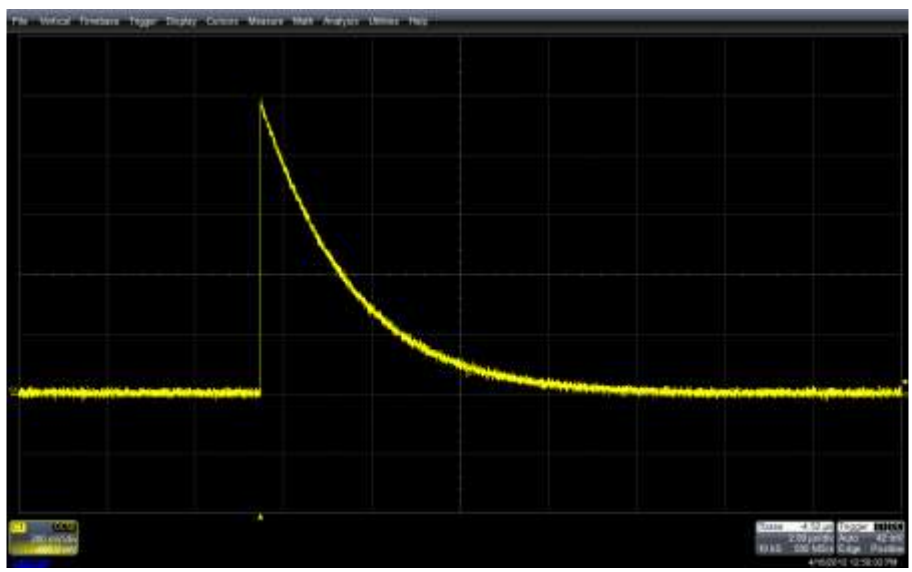

Figure 1: An exponential signal

$$V(t) = ae^{-t/\tau} + b$$

where:

a and b are arbitrary constants

τ is the exponential time constant

and $t \geq 0$.

Our signal is a little noisy. We can reduce the noise by applying a noise filter on channel 1. This applies a finite impulse response digital low pass filter with a user selectable bandwidth. This is an important step as we will be applying this signal to a logarithmic math trace and we want to eliminate noise spikes that might be negative.

Figure 2: Reducing the noise level on the acquired signal

To determine the exponential time constant, $-\tau$, we can take the natural logarithm of both sides of the original exponential expression and solve for τ:

$$\ln [(V(t)-b)/a] = -t/\tau$$

$$\tau = -t / \ln[(V(t)-b)/a]$$

If we normalize the waveform so that a = 1 and b = 0
We can make the time constant τ **= - t/ln[V(t)].**

If we set $t = \infty$ then $V(\infty) = b$;: this is the signal's baseline value

Similarly, at t=0 V(0) = a + b ; this is the amplitude near the trigger point which is the maximum amplitude for the waveform.

In Figure 3 we measure the value of the trace by using the top and base parameters.

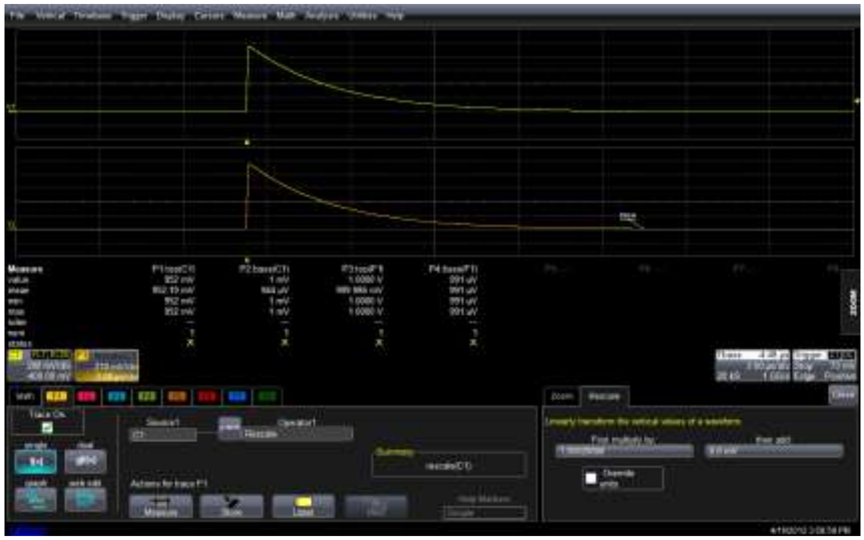

Figure 3: Using the rescale function to normalize the exponential waveform to unity gain

From figure 3 we see that the constant b is nominally 0. Based on that determination the constant a= 952.2 mV. To normalize this waveform to unity amplitude we need to rescale the trace by multiplying by 1/0.9522 = 1.0502. We can do this in the math function rescale as shown in Figure 4. In this figure parameter P1 and P2 read the top and base of the input waveform. P3 and P4 read the top and base of the rescaled waveform.

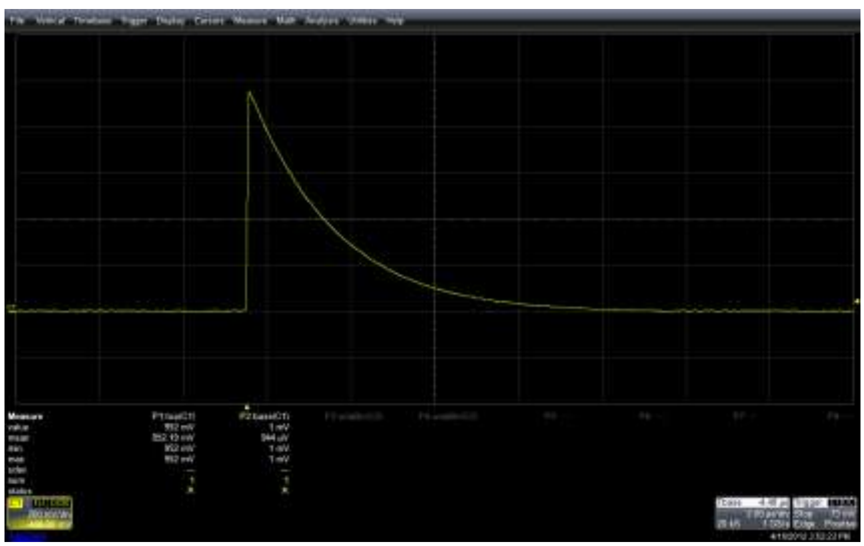

Figure 4: Measuring the top and base to determine the constants a and b

The rescaled waveform is now unipolar positive with a base of zero and a top value of 1 V. If we apply this to the natural logarithm (base e) math function the slope of this waveform is the reciprocal of the time constant, **τ.**

Figure 5 shows both the rescaled waveform and the natural logarithm of that waveform. Note that the logarithmic scaling renders the exponential function as linear.

Figure 5: Using the natural logarithm(Ln) math function to linearize the exponential function. The slope of the linear section of the Ln trace is the reciprocal of the exponential time constant

We can use the relative time cursors to read the slope of the Ln waveform. We can obtain the best accuracy by measuring over the linear portion of the trace as shown in Figure 6. The cursors read the slope as -495.7kV/s. The time constant, τ, is the reciprocal of this number or 2.017 μs.

Figure 6: Measuring the slope of the Ln function to determine the time constant

So the original waveform can be described as: $\mathbf{v(t)= 0.9522\ e^{-t/(2*10^{\wedge}-6)}}$.